Math 2D Multi-Variable Calculus Homework Questions 5

14.6 Directional Derivatives and the Gradient Vector

8, 10 (a) Find the gradient of f.

(b) Evaluate the gradient at the point P.

(c) Find the rate of change of f at P in the direction of the vector $\mathbf{u}$.

8 * $f(x,y) = y^2/x, \qquad P(1,2), \qquad \mathbf{u} = \frac{1}{3}(2\mathbf{i} + \sqrt{5}\mathbf{j})$

10 $f(x,y,z) = y^2 e^{xyz}, \qquad P(0,1,-1), \qquad \mathbf{u} = \frac{1}{13}(3\mathbf{i} + 4\mathbf{j} + 12\mathbf{k})$

12, 16 Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

12 * $f(x,y) = \dfrac{x}{x^2+y^2}, \qquad (1,2), \qquad \mathbf{v} = 3\mathbf{i} + 5\mathbf{j}$

16 $f(x,y,z) = \sqrt{xyz}, \qquad (3,2,6), \qquad \mathbf{v} = -\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$

32 * The temperature at a point (x,y,z) is given by

$$T(x,y,z) = 200e^{-x^2-3y^2-9z^2}$$

where T is measured in °C and x, y, z in meters.

(a) Find the rate of change of temperature at the point $P(2,-1,2)$ in the direction toward the point $(3,-3,3)$.

(b) In which direction does the temperature increase fastest?

(c) Find the maximum rate of increase at P.

42, 46 Find the equations of (a) the tangent plane and (b) the normal line to the given surface at the specified point.

42 * $y = x^2 - z^2, \quad (4,7,3)$

46 $x^4 + y^4 + z^4 = 3x^2y^2z^2, \quad (1,1,1)$

14.7 Maximum and Minimum Values

6–18 Find the local maximum and minimum values and saddle point(s) of the function.

6 $f(x,y) = xy - 2x - 2y - x^2 - y^2$

10 * $f(x,y) = xy(1-x-y)$

14 * $f(x,y) = y\cos x$

18 $f(x,y) = \sin x \sin y, \qquad -\pi < x < \pi, \qquad -\pi < y < \pi$

30, 34 Find the absolute maximum and minimum values of f on the set D.

30 * $f(x,y) = x + y - xy$, D is the closed triangular region with vertices $(0,0)$, $(0,2)$, and $(4,0)$.

34 $f(x,y) = xy^2$, $D = \{(x,y) \mid x \geq 0,\ y \geq 0,\ x^2 + y^2 \leq 3\}$

42 Find the points on the surface $y^2 = 9 + xz$ that are closest to the origin.

48 Find the dimensions of the rectangular box with largest volume if the total surface area is $64\,\text{cm}^2$.

56 * Find an equation of the plane that passes through the point $(1,2,3)$ and cuts off the smallest volume in the first octant.