

Math 2E Suggested Written Questions 16.6

16.6 Parametric Surfaces and their Areas

1. Identify the surface with the given vector equation.

(a) $\mathbf{r}(u, v) = 2 \sin u \mathbf{i} + 3 \cos u \mathbf{j} + v \mathbf{k}$

(b) $\mathbf{r}(s, t) = s \sin 2t \mathbf{i} + s^2 \mathbf{j} + s \cos 2t \mathbf{k}$

2. Find a parametric representation of the surface.

(a) The plane that passes through the point $(0, -1, 5)$ and contains the vectors $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$.

(b) The part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz -plane.

(c) The part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = -2$ and $z = 2$.

(d) The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$.

3. Find parametric equations for the surface obtained by rotating the curve $x = y^2 - \frac{1}{4}y^4$, $-2 \leq y \leq 2$ about the y -axis. Now sketch the surface and some of its grid lines.

4. Find an equation of the tangent plane to the given parametric surface at the specified point.

(a) $x = u^2 + 1$, $y = v^3 + 1$, $z = u + v$; $(5, 2, 3)$.

(b) $\mathbf{r}(u, v) = \sin u \mathbf{i} + \cos u \sin v \mathbf{j} + \sin v \mathbf{k}$; $u = v = \frac{\pi}{6}$.

5. Find an equation of the tangent plane to the parametric surface defined by

$$\mathbf{r}(u, v) = (1 - u^2 - v^2) \mathbf{i} - v \mathbf{j} - u \mathbf{k}$$

at the point $(-1, -1, -1)$. Graph the surface and the tangent plane.

6. Find the area of each surface.

(a) The part of the plane with vector equation

$$\mathbf{r}(u, v) = (u + v) \mathbf{i} + (2 - 3u) \mathbf{j} + (1 + u - v) \mathbf{k}, \quad 0 \leq u \leq 2, \quad -1 \leq v \leq 1$$

(b) The part of the surface $z = 1 + 3x + 2y^2$ lying above the triangle with vertices $(0, 0)$, $(0, 1)$ and $(2, 1)$.

(c) The part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

(d) The upper part of the sphere $x^2 + y^2 + z^2 = b^2$ that lies inside the cylinder $x^2 + y^2 = a^2$, where $0 < a < b$.

7. Suppose that $\mathbf{r}(u, v)$ is a regular surface and that $u = u(\alpha, \beta)$ and $v = v(\alpha, \beta)$ for some alternative co-ordinates (α, β) . Prove that

$$|\mathbf{r}_u \times \mathbf{r}_v| = \left| \frac{\partial(\alpha, \beta)}{\partial(u, v)} \right| |\mathbf{r}_\alpha \times \mathbf{r}_\beta|$$

Hence conclude that the area element dS is independent of parametrization.

8. The figure shows the surface created when the cylinder $y^2 + z^2 = 1$ intersects the cylinder $x^2 + z^2 = 1$. Find the area of this surface.

