Math 2E Multi-Variable Calculus Homework Questions 1

15 Multiple Integrals

15.2 Iterated Integrals

4 Calculate the iterated integral $\int_0^1 \int_1^2 (4x^3 - 9x^2y^2)\,\mathrm{d}y\,\mathrm{d}x$.

8 Calculate the iterated integral $\int_1^3 \int_1^5 \frac{\ln y}{xy}\,\mathrm{d}y\,\mathrm{d}x$.

16 Calculate the double integral $\iint_R y + xy^{-2}\,\mathrm{d}A$ where $R = \{(x,y) : 0 \le x \le 2, 1 \le y \le 2\}$.

20 Calculate the double integral $\iint_R \frac{x}{1+xy}\,\mathrm{d}A$ where $R = [0,1] \times [0,1]$.

24 Sketch the solid whose volume is given by the iterated integral $\int_0^1 \int_0^1 (2 - x^2 - y^2)\,\mathrm{d}y\,\mathrm{d}x$.

28 Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = \pm 1, y = 0, y = \pi$ and $z = 0$.

15.3 Double Integrals over General Regions

4 Evaluate the iterated integral $\int_0^2 \int_y^{2y} xy\,\mathrm{d}x\,\mathrm{d}y$.

8 Evaluate the double integral $\iint_R \frac{y}{x^5+1}\,\mathrm{d}A$ where $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le x^2\}$.

16 Set up iterated integrals for both orders of integration. Then evaluate the double integral using the easier method:

$$\iint_D y^2 e^{xy}\,\mathrm{d}A, \text{ where } D \text{ is bounded by } y = x, y = 4, x = 0.$$

20 Evaluate the double integral $\iint_D xy^2\,\mathrm{d}A$ where D is enclosed by $x = 0$ and $x = \sqrt{1 - y^2}$.

22 Evaluate the double integral $\iint_D 2xy\,\mathrm{d}A$ where D is the triangular region with vertices $(0,0)$, $(1,2)$ and $(0,3)$.

26 Find the volume of the solid enclosed by the paraboloid $z = x^2 + 3y^2$ and the planes $x = 0, y = 1, y = x$ and $z = 0$.

46 Sketch the region of integration and change the order of integration:

$$\int_{-2}^{2} \int_0^{\sqrt{4-y^2}} f(x,y)\,\mathrm{d}x\,\mathrm{d}y.$$

52 Evaluate the integral by changing the order of integration:

$$\int_0^1 \int_x^1 e^{x/y}\,\mathrm{d}y\,\mathrm{d}x.$$

15.4 Double Integrals in Polar Co-ordinates

6 Sketch the region whose area is given by the integral $\int_{\pi/2}^{\pi} \int_{0}^{2\sin\theta} r \, \mathrm{d}r \, \mathrm{d}\theta$ and evaluate the area.

10 By changing to polar co-ordinates, evaluate the integral $\iint_R \frac{y^2}{x^2+y^2} \, \mathrm{d}A$, where R is the region lying between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ for $0 < a < b$.

12 By changing to polar co-ordinates, evaluate the integral $\iint_D \cos\sqrt{x^2 + y^2} \, \mathrm{d}A$, where D is the disk with center the origin and radius 2.

20 Use polar co-ordinates to find the volume of the solid lying below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.

26 Use polar co-ordinates to find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

28 (a) A cylindrical drill of radius r_1 is used to bore a hole through the center of a sphere of radius r_2. Find the volume of the ring-shaped solid that remains.

(b) Express the volume in part (a) in terms of the height h of the ring. Notice that the volume depends only on h, not on r_1 or r_2.

38 Let D be the disk with center the origin and radius a. What is the average distance from points in D to the origin?

15.7 Triple Integrals

6 Evaluate the iterated integral $\int_0^1 \int_0^1 \int_0^{\sqrt{1-z^2}} \frac{z}{y+1} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z$.

12 Evaluate the triple integral $\iiint_E \sin y \, \mathrm{d}V$, where E lies below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$ and $(0, \pi, 0)$.

16 Evaluate the triple integral $\iiint_T xyz \, \mathrm{d}V$, where T is the solid tetrahedron with vertices (0,0,0), (1,0,0), (1,1,0) (1,0,1).

22 Use a triple integral to evaluate the volume of the solid enclosed by the cylinder $x^2 + z^2 = 4$ and the planes $y = -1$ and $y + z = 4$.