Math 2E Multi-Variable Calculus Homework Questions 3

16.5 Curl and Divergence

2–8 Find (a) the curl and (b) the divergence of the vector field

2 $\mathbf{F}(x,y,z) = xy^2z^3\mathbf{i} + x^3yz^2\mathbf{j} + x^2y^3z\mathbf{k}$

4 $\mathbf{F}(x,y,z) = \sin yz\mathbf{i} + \sin zx\mathbf{j} + \sin xy\mathbf{k}$

6 $\mathbf{F}(x,y,z) = e^{xy}\sin z\mathbf{j} + y\tan^{-1}(x/z)\mathbf{k}$

8 $\mathbf{F}(x,y,z) = xy^{-1}\mathbf{i} + yz^{-1}\mathbf{j} + zx^{-1}\mathbf{k}$

12 Let f be a scalar field and $\mathbf{F}$ a vector field. State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar field.

(a) $\operatorname{curl} f$
(b) $\operatorname{grad} f$
(c) $\operatorname{div} \mathbf{F}$
(d) $\operatorname{curl}(\operatorname{grad} f)$
(e) $\operatorname{grad} \mathbf{F}$
(f) $\operatorname{grad}(\operatorname{div} \mathbf{F})$
(g) $\operatorname{div}(\operatorname{grad} f)$
(h) $\operatorname{grad}(\operatorname{div} f)$
(i) $\operatorname{curl}(\operatorname{curl} \mathbf{F})$
(j) $\operatorname{div}(\operatorname{div} \mathbf{F})$
(k) $(\operatorname{grad} f) \times (\operatorname{div} \mathbf{F})$
(l) $\operatorname{div}(\operatorname{curl}(\operatorname{grad} f)$

14, 16 Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

14 $\mathbf{F}(x,y,z) = xyz^2\mathbf{i} + x^2yz^2\mathbf{j} + x^2y^2z\mathbf{k}$

16 $\mathbf{F}(x,y,z) = \mathbf{i} + \sin z\mathbf{j} + y\cos z\mathbf{k}$

20 Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\operatorname{curl}\mathbf{G} = xyz\mathbf{i} - y^2z\mathbf{j} + yz^2\mathbf{k}$? Explain.

26 Prove that $\operatorname{curl}(f\mathbf{F}) = f\operatorname{curl}\mathbf{F} + \nabla f \times \mathbf{F}$

28 Prove that $\operatorname{div}(\nabla f \times \nabla g) = 0$

30, 32 Let $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $r\,|\mathbf{r}|$

30 Verify each identity:

(a) $\nabla \cdot \mathbf{r} = 3$
(b) $\nabla \cdot (r\mathbf{r}) = 4r$
(c) $\nabla^2 r^3 = 12r$

32 If $\mathbf{F} = r^{-p}\mathbf{r}$, find $\operatorname{div}\mathbf{F}$. Is there a value of p for which $\operatorname{div}\mathbf{F} = 0$?

16.6 Parametric Surfaces and their Areas

4, 6 Identify the surface with the given vector equation

4 $\mathbf{r}(u,v) = 2\sin u\mathbf{i} + 3\cos u\mathbf{j} + v\mathbf{k}, \qquad 0 \le v \le 2$

6 $\mathbf{r}(s,t) = s\sin 2t\mathbf{i} + s^2\mathbf{j} + s\cos 2t\mathbf{k}$

20–26 Find a parametric representation of the surface

20 The plane that passes through the point $(0,-1,5)$ and contains the vectors $(2,1,4)$ and $(-3,2,5)$

22 The part of the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ that lies to the left of the xz-plane

24 The part of the sphere $x^2 + y^2 + z^2 = 16$ that lies between the planes $z = -2$ and $z = 2$

26 The part of the plane $z = x + 3$ that lies inside the cylinder $x^2 + y^2 = 1$

30 Find parametric equations for the surface obtained by rotating the curve $x = 4y^2 - y^4$, $-2 \leq y \leq 2$ about the y-axis and use them to graph the surface

34, 36 Find an equation of the tangent plane to the given parametric surface at the specified point

34 $x = u^2 + 1, \quad y = v^3 + 1, \quad z = u + v; \quad (5, 2, 3)$

36 $\mathbf{r}(u, v) = \sin u\mathbf{i} + \cos u \sin v\mathbf{j} + \sin v\mathbf{k}; \quad u = \pi/6, \quad v = \pi/6$

38 Find an equation of the tangent plane to the parametric surface defined by

$$\mathbf{r}(u, v) = (1 - u^2 - v^2)\mathbf{i} - v\mathbf{j} - u\mathbf{k}$$

at the point $(-1, -1, -1)$. Graph the surface and the tangent plane.

40–50 Find the area of the surface

40 The part of the plane with vector equation

$$\mathbf{r}(u, v) = (u + v)\mathbf{i} + (2 - 3u)\mathbf{j} + (1 + u - v)\mathbf{k}, \quad 0 \leq u \leq 2, \quad -1 \leq v \leq 1$$

44 The part of the surface $z = 1 + 3x + 2y^2$ lying above the triangle with vertices $(0, 0)$, $(0, 1)$ and $(2, 1)$

46 The part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$

50 The part of the sphere $x^2 + y^2 + z^2 = b^2$ that lies inside the cylinder $x^2 + y^2 = a^2$, where $0 < a < b$

60 (a) Show that the parametric equations $x = a \cosh u \cos v$, $y = b \cosh u \sin v$, $z = c \sinh u$ represent a hyperboloid of one sheet.

(b) Use the parametric equations in part (a) to graph the hyperboloid for the case $a = 1$, $b = 2$, $c = 3$.

(c) Set up, but do not evaluate, a double integral for the surface area of the hyperboloid in part (b) that lies between the planes $z = -3$ and $z = 3$.

62 The figure shows the surface created when the cylinder $y^2 + z^2 = 1$ intersects the cylinder $x^2 + z^2 = 1$. Find the area of this surface.