Math 2E: Pre-req Quiz

1. Evaluate the integrals:

   (a) $\displaystyle\int (6x+1)\sqrt{3x^2+x}\,\mathrm{d}x.$

   (b) $\displaystyle\int_0^{\pi/4} \frac{\sin x}{\cos x}\,\mathrm{d}x.$

   (c) $\displaystyle\int \frac{2}{x^2-4x-32}\,\mathrm{d}x.$

   (d) $\displaystyle\int_1^{e^2} t^2 \ln t\,\mathrm{d}t.$

2. The arc-length of a curve $y = f(x)$ between $x = a$ and $x = b$ is given by $\int_a^b \sqrt{1+(f'(x))^2}\,\mathrm{d}x$. Compute the arc-length of $y = x^{3/2}$ for $0 \le x \le 4$.

3. Find the unit tangent vector to the curve $\mathbf{v}(t) = \begin{pmatrix} t^2 \\ 4t \\ 4\ln t \end{pmatrix}$ at $t = 3$.

4. Identify each equation with one of the following surfaces: plane, cone, cylinder, paraboloid, ellipsoid, hyperboloid of one sheet, hyperboloid of two sheets. Each type of surface appears exactly once.

   (a) $x^2+y^2=7$.

   (b) $x^2+y^2=z$.

   (c) $x^2+\dfrac{y^2}{4}+\dfrac{z^2}{9}=1$.

   (d) $2x+7y-3z=2$.

   (e) $3x^2+7y^2=z^2$.

   (f) $\dfrac{x^2}{16}+\dfrac{y^2}{4}-\dfrac{z^2}{9}=1$.

   (g) $\dfrac{x^2}{16}-\dfrac{y^2}{4}-\dfrac{z^2}{9}=1$.

   You should be able to do this by vizualizing the cross-sections (level-curves) of each surface with respect to each of the co-ordinate planes.

5. Find the gradient of the function $f(x,y) = x^4+y^4-4x-32y+10$. Find its unique critical point, and use the Second Derivative Test to decide whether the critical point is a local minimum, local maximum, or a saddle point.