Math 2J Midterm 1 answers

1/ $$(A-2B)^2 = (A-2B)(A-2B) = A^2 - 2AB - 2BA + 4B^2$$
$$\neq A^2 - 4AB + 4B^2$$
since $AB \neq BA$ in general.

2/ a) $$\left(\begin{array}{cccc|c} 1 & -2 & 1 & -2 & 4 \\ 2 & -4 & 0 & -6 & 2 \\ 2 & -4 & -1 & -7 & -1 \\ 3 & -6 & -1 & -10 & 0 \end{array}\right) \rightsquigarrow \begin{array}{c} \\ R_2-2R_1 \\ R_3-2R_1 \\ R_4-3R_1 \end{array} \left(\begin{array}{cccc|c} 1 & -2 & 1 & -2 & 4 \\ 0 & 0 & -2 & -2 & -6 \\ 0 & 0 & -3 & -3 & -9 \\ 0 & 0 & -4 & -4 & -12 \end{array}\right)$$

$$\rightsquigarrow \begin{array}{c} R_1+\frac{1}{2}R_2 \\ \div -2 \\ R_3-\frac{3}{2}R_2 \\ R_4-2R_2 \end{array} \left(\begin{array}{cccc|c} 1 & -2 & 0 & -3 & 1 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftarrow \text{Reduced Row Echelon Form.}$$

b) lead x_1, x_3, free x_2, x_4

c) $$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}.$$

3/ $$A^{-1} = \left(\begin{array}{c|c} I_p & -B \\ \hline 0 & I_{n-p} \end{array}\right)$$

4/ $$\det A = 7\begin{vmatrix} -6 & 15 \\ 1 & -1 \end{vmatrix} - 2\begin{vmatrix} 3 & 15 \\ 1 & -1 \end{vmatrix} + 3\begin{vmatrix} 3 & -6 \\ 1 & 1 \end{vmatrix}$$

$$= 7\cdot(-9) - 2\cdot(-18) + 3\cdot 9$$
$$= 9(-7+4+3) = 0.$$

A singular $\Rightarrow A\underline{x} = 0$ has a non-zero solution $\hat{\underline{x}}$

$\Rightarrow A\underline{x} = 0$ has ∞ solutions of which $\lambda\hat{\underline{x}} : \lambda \in \mathbb{R}$ are.

(In fact all solutions are multiples of $\hat{\underline{x}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$)

5/ $$(A|I) = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 2 & 2 & 0 & 0 & 1 \end{array}\right) \overset{\begin{array}{l} R_2+R_1 \\ R_3-R_1 \end{array}}{\rightsquigarrow} \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 4 & 6 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$$

$$\overset{\begin{array}{l} R_1+3R_3 \\ R_2+6R_3 \\ \times -1 \end{array}}{\rightsquigarrow} \left(\begin{array}{ccc|ccc} 1 & 2 & 0 & -2 & 0 & 3 \\ 0 & 4 & 0 & -5 & 1 & 6 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$$

$$\overset{\begin{array}{l} R_1-\frac{1}{2}R_2 \\ \div 4 \end{array}}{\rightsquigarrow} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 0 & -5/4 & 1/4 & 3/2 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{pmatrix} 2 & -2 & 0 \\ -5 & 1 & 6 \\ 4 & 0 & -4 \end{pmatrix}$$

6/a/ $A(\operatorname{adj} A) = (\det A)\, I$

$$\Rightarrow \det A \cdot \det(\operatorname{adj} A) = (\det A)^n$$

$$\Rightarrow \det(\operatorname{adj} A) = (\det A)^{n-1}.$$

$\left\{ \begin{array}{l} \text{since } \det(PQ) = \det P \cdot \det Q \\ \text{and } \det(\alpha P) = \alpha^n \det P \end{array}\right.$

b/ $n = 4 \Rightarrow \left.\begin{array}{l} \underset{\substack{\parallel \\ 27}}{\det(\operatorname{adj} B)} = (\det B)^3 \end{array}\right\} \Rightarrow \det B = 3.$