Math 2J: Linear Algebra & Infinite Series Midterm v2

Total 50 marks: marks per question are in brackets. You must show working for calculations, merely stating the answer will get you no marks

1. Let A, B be $n \times n$ matrices. Is it true that

$$(3A + B)^2 = 9A^2 + 6AB + B^2?$$

   If not, what should the right hand side be? (5)

2. Consider the following system of equations

$$\begin{cases} x_1 - 3x_2 + x_3 + 3x_4 = 3, \\ -x_1 + 3x_2 + 2x_3 = 6, \\ 2x_1 - 6x_2 + 5x_3 + 9x_4 = 15, \\ -2x_1 + 6x_2 + 2x_3 - 2x_4 = 6. \end{cases}$$

   (a) Find the Reduced Row Echelon form of the augmented matrix of the system. (8)
   (b) List the lead and free variables. (2)
   (c) Write down all the solutions to the system. (3)

3. Suppose that the $n \times n$ matrix A has block form

$$A = \left(\begin{array}{c|c} I_p & 0 \\ \hline B & I_{n-p} \end{array} \right),$$

   where B is some $(n - p) \times p$ matrix and the O is the $p \times (n - p)$ zero-matrix. Find the inverse of A in block form. (5)

4. Calculate the determinant of the matrix

$$A = \begin{pmatrix} 3 & 1 & 7 \\ 9 & -6 & 12 \\ 1 & 2 & 4 \end{pmatrix}.$$

   Hence or otherwise, deduce the number of solutions to the system $A\mathbf{x} = 0$. Justify your answer. (*You do not have to find the solutions!*) (7)

5. Let A be the matrix

$$A := \begin{pmatrix} 1 & 3 & 2 \\ -1 & 3 & 2 \\ 1 & 3 & 1 \end{pmatrix}.$$

   Find the inverse of A by using the method of row operations applied to the augmented matrix $(A|I)$. (10)

6. The *adjoint* matrix adj A of an $n \times n$ matrix A satisfies the matrix equation $A(\text{adj } A) = (\det A)I$.

   (a) Prove that $\det(\text{adj } A) = (\det A)^{n-1}$. (7)
   (b) Suppose that B is a 4×4 matrix whose adjoint has determinant -8. What is the determinant of B? (3)