Math 2J Midterm v2 Answers

1/ $(3A+B)^2 = (3A+B)(3A+B) = 9A^2+3AB+3BA+B^2$
$\neq 9A^2+6AB+B^2$
since $AB \neq BA$ in general.

2/(a) $\left(\begin{array}{cccc|c} 1 & -3 & 1 & 3 & 3 \\ -1 & 3 & 2 & 0 & 6 \\ 2 & -6 & 5 & 9 & 15 \\ -2 & 6 & 2 & -2 & 6 \end{array}\right) \xrightarrow{\substack{R_2+R_1 \\ R_3-2R_1 \\ R_4+2R_1}} \left(\begin{array}{cccc|c} 1 & -3 & 1 & 3 & 3 \\ 0 & 0 & 3 & 3 & 9 \\ 0 & 0 & 3 & 3 & 9 \\ 0 & 0 & 4 & 4 & 12 \end{array}\right)$

$\xrightarrow{\substack{R_1-\frac{1}{3}R_2 \\ \div 3 \\ R_3-R_2 \\ R_4-\frac{4}{3}R_2}} \left(\begin{array}{cccc|c} 1 & -3 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right) \leftarrow$ Reduced Row Echelon Form.

b/ lead x_1, x_3, free x_2, x_4

$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \quad \lambda, \mu \in \mathbb{R}.$

3/ $A^{-1} = \left(\begin{array}{c|c} I_p & 0 \\ \hline -B & I_{n-p} \end{array}\right)$

4/ $\det A = 3 \begin{vmatrix} -6 & 12 \\ 2 & 4 \end{vmatrix} - \begin{vmatrix} 9 & 12 \\ 1 & 4 \end{vmatrix} + 7 \begin{vmatrix} 9 & -6 \\ 1 & 2 \end{vmatrix}$

$= 3(-48) - 24 + 7 \cdot 24$

$= 24(-6-1+7) = 0$

A singular $\Rightarrow A\underline{x} = 0$ has a non-zero solution $\hat{\underline{x}}$.

$\Rightarrow A\underline{x} = 0$ has ∞ solutions ($\lambda \hat{\underline{x}} : \lambda \in \mathbb{R}$ being some)

(In fact all solutions are multiples of $\hat{\underline{x}} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$)

5/ $(A|I) = \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ -1 & 3 & 2 & 0 & 1 & 0 \\ 1 & 3 & 1 & 0 & 0 & 1 \end{array}\right) \rightsquigarrow \begin{array}{c} \\ R_2+R_1 \\ R_3-R_1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 6 & 4 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 1 \end{array}\right)$

$\rightsquigarrow \begin{array}{c} R_1+2R_3 \\ R_2+4R_3 \\ \times -1 \end{array} \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & -1 & 0 & 2 \\ 0 & 6 & 0 & -3 & 1 & 4 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$

$\rightsquigarrow \begin{array}{c} R_1-\frac{1}{2}R_2 \\ \div 6 \\ \\ \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 0 & -1/2 & 1/6 & 2/3 \\ 0 & 0 & 1 & 1 & 0 & -1 \end{array}\right)$

$\Rightarrow A^{-1} = \frac{1}{6}\begin{pmatrix} 3 & -3 & 0 \\ -3 & 1 & 4 \\ 6 & 0 & -6 \end{pmatrix}$

6/ a/ $A(\operatorname{adj} A) = (\det A)\, I$

$\Rightarrow \det A \cdot \det(\operatorname{adj} A) = (\det A)^n$

$\Rightarrow \det(\operatorname{adj} A) = (\det A)^{n-1}$

$\left\{ \begin{array}{l} \text{since } \det(PQ) = \det P \cdot \det Q \\ \text{and } \det(\alpha P) = \alpha^n \det P. \end{array} \right.$

b/ $n = 4 \Rightarrow \underbrace{\det(\operatorname{adj} B)}_{\substack{\parallel \\ -8}} = (\det B)^3$

$\Rightarrow \det B = -2$