

Math 3A: Homework 5

Submit these questions at the discussion on Tuesday 22nd November

1. Find the eigenvalues and eigenspaces of the following matrices.

(a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & -8 \\ 1 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 3 \\ 1 & 1 & -1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$

2. For each of the matrices A in question 1, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Use this diagonalization to compute A^5 .

3. Show that any matrix of the form

$$A = \begin{pmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & b \end{pmatrix}$$

is defective.

4. For what values of α is the following matrix defective?

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix}$$

5. Prove that A is singular if and only if A has zero as an eigenvalue.

6. Suppose that λ is an eigenvalue of A and that A is invertible. Prove that λ^{-1} is an eigenvalue of A^{-1} .

7. Suppose that λ is an eigenvalue of A and that $B = (A - I)^2$.

(a) Prove that $(\lambda - 1)^2$ is an eigenvalue of B .

(b) Suppose that 1 is an eigenvalue of A . Prove that B is singular.

8. A matrix A is *idempotent* if $A^2 = A$. Prove that the only eigenvalues of an idempotent matrix are 0 or 1.

9. (a) Prove that A and A^T have the same eigenvalues.

(b) Give an example which shows that A and A^T need not have the same eigenvectors.