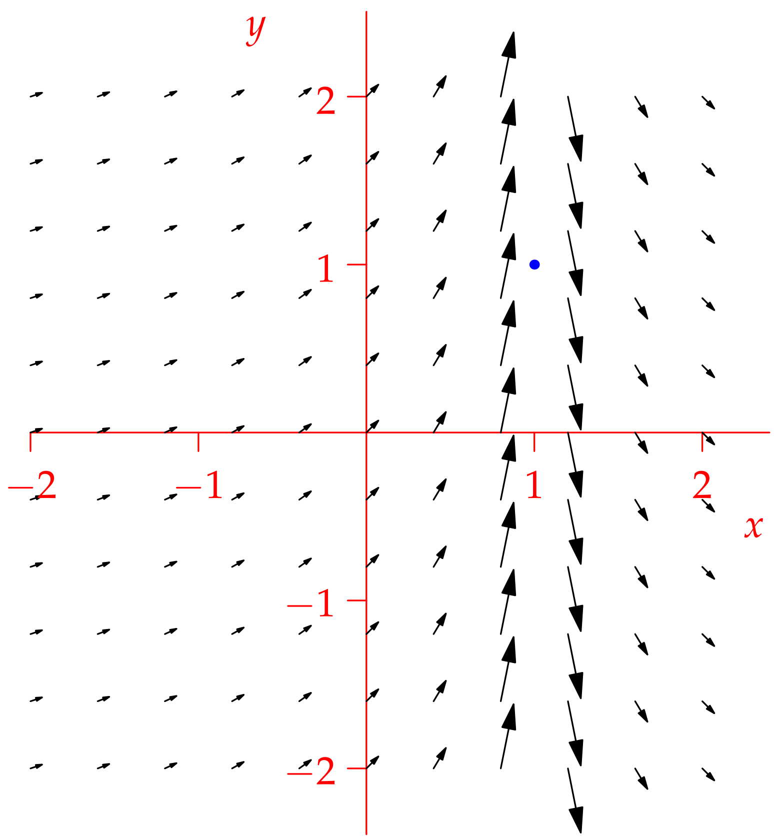
Existence and Uniqueness (Picard's Theorem)

In each case the theorem *does not apply*

$$\begin{cases} \frac{dy}{dx} = \frac{1}{1-x} \\ y(1) = 1 \end{cases} \quad \text{has no solutions}$$

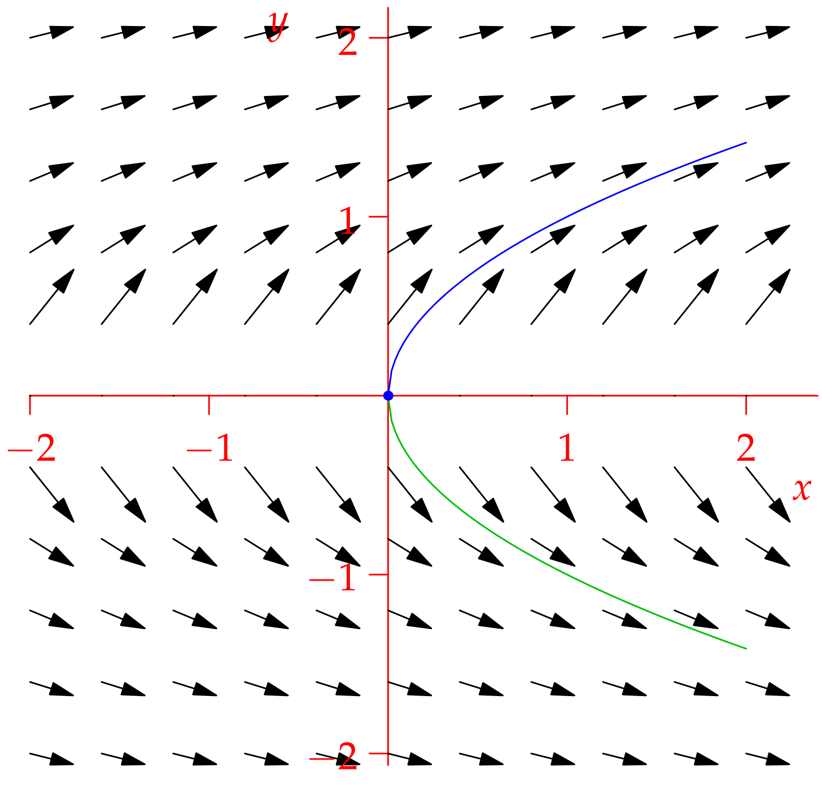
$f(x, y) = \frac{1}{1-x}$ is not defined (let alone continuous) at $(x, y) = (1, 1)$



Dot doesn't know whether to travel up or down!

$$\begin{cases} \frac{dy}{dx} = \frac{1}{2y} \\ y(0) = 0 \end{cases} \quad \text{has two solutions } y(x) = \pm\sqrt{x}$$

$$f(x,y) = \frac{1}{2y} \text{ is not defined at } (x,y) = (0,0)$$

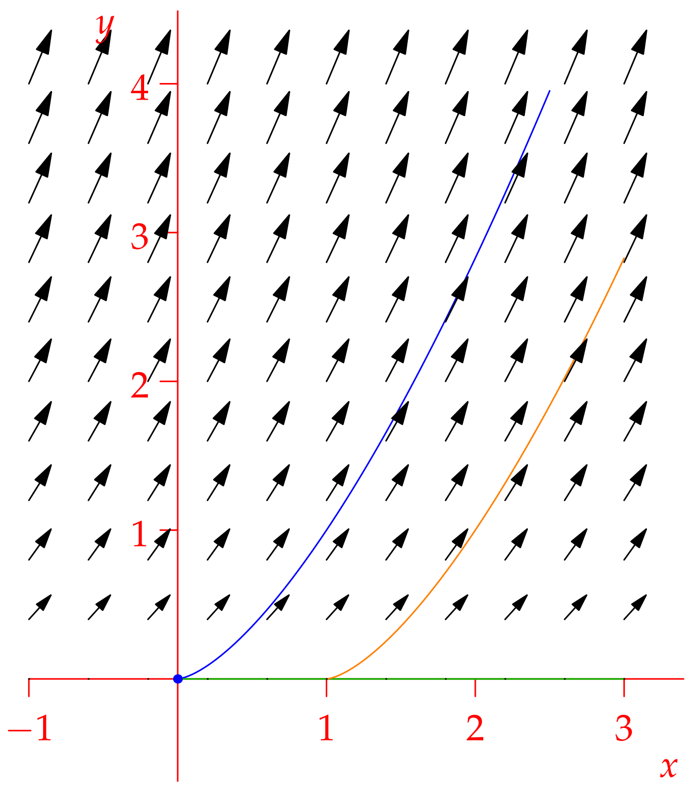


Dot could move up or down from origin

$$\begin{cases} \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{2}y^{1/3} \\ y(0) = 0 \end{cases} \quad \text{has infinitely many solutions}$$

$f(x,y) = \frac{3}{2}y^{1/3}$ is continuous at $(x,y) = (0,0)$, but its y-derivative is not:

$$\frac{\partial f}{\partial y} = \frac{1}{2}y^{-2/3}$$



$y(x) \equiv 0$ is a solution, as is any function of the form

$$y(x) = \begin{cases} 0 & x < a \\ (x-a)^{3/2} & x \geq a \end{cases} \quad \text{for any constant } a \geq 0$$