

Math 3D Differential Equations Homework Questions 4

1. Write the system in the form $\mathbf{x}' = A(t)\mathbf{x} + \mathbf{f}(t)$.

$$(a) \begin{cases} x' = tx - e^t y + \cos t \\ y' = e^{-t} x + t^2 y - \sin t \end{cases}$$

$$(b) \begin{cases} x' = tx - y + e^t z \\ y' = 2x + t^2 y - z \\ z' = 6y - 7z \end{cases}$$

2. You are given the following:

$$\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -3 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}_1(t) = e^{3t} \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad \mathbf{x}_2(t) = e^{-2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad x(0) = 0, \quad y(0) = 5$$

- Verify that the given vectors are solutions of the system.
- Show that the solutions are linearly independent.
- Find the general solution to the system.
- Find the particular solution satisfying the given initial conditions.

3. Repeat the previous question for the following:

$$\mathbf{x}' = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 3 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}_1(t) = e^t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2(t) = e^{3t} \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{x}_3(t) = e^{5t} \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix},$$
$$x(0) = 0, \quad y(0) = 0, \quad z(0) = 4$$

4. Solve the following system of equations using both the eigenvalue method and the method of elimination.

$$\begin{cases} x' = 9x + 5y \\ y' = -6x - 2y \end{cases} \quad x(0) = 1, \quad y(0) = 0$$

5. Repeat the previous question for

$$\begin{cases} x' = 3x - 4y \\ y' = 4x + 3y \end{cases}$$

6. Use the eigenvalue method to solve

$$\begin{cases} x' = 3x + 2y + 2z \\ y' = -5x - 4y - 2z \\ z' = 5x + 5y + 3z \end{cases}$$

7. Use the eigenvalue/hybrid method to solve the system $\mathbf{x}' = A\mathbf{x}$, where

$$A = \begin{pmatrix} -5 & 3 & 1 \\ -14 & 7 & 6 \\ 3 & -1 & -3 \end{pmatrix}$$

Hint: you should get simple imaginary eigenvalues. Find the eigenvectors and convert the solutions to real form using Euler's formula. Also relax, nothing this tough will be on an exam :)

8. Find the particular solution of the system

$$\frac{dx}{dt} = 3x + z \quad \frac{dy}{dt} = 9x - y + 2z \quad \frac{dz}{dt} = -9x + 4y - z$$

that satisfies the initial conditions $x(0) = 3, y(0) = 5, z(0) = 6$. You will again get complex eigenvalues.