Math 3D Differential Equations Homework Questions 5

1. Apply the definition to find directly the Laplace transform of $f(t) = \sin^2 t$
2. Use the table of transforms to find the Laplace transform of $f(t) = \sin 2t + \cos 2t$
3. Use the table of transforms to find the Inverse Laplace transform of $F(s) = \frac{3s+1}{s^2+4}$
4. Use the integral

   $$\int e^{ax} \cos bx \, \mathrm{d}x = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$$

   to obtain $\mathcal{L}\{\cos kt\}$ directly from the definition of the Laplace transform.
5. Apply the translation theorem to find the Laplace transform of $f(t) = t^5 e^{-4t}$
6. Use partial fractions to find the inverse Laplace transforms
   (a) $F(s) = \frac{5s-6}{s^2-3s}$
   (b) $F(s) = \frac{1}{s^4-16}$
7. Use Laplace transforms to solve the given initial value problem
   (a) $x'' + 9x = 0 \quad x(0) = 3 \quad x'(0) = 4$
   (b) $x'' + 4x = \cos t \quad x(0) = 0 \quad x'(0) = 0$
   (c) $x'' - 4x = 3t \qquad x(0) = x'(0) = 0$
8. (Harder) Apply the Laplace transform method to solve the *system*

   $$\begin{cases} x'' + 2x + 4y = 0 \\ y'' + x + 2y = 0 \end{cases} \qquad x(0) = y(0) = 0 \quad x'(0) = y'(0) = -1$$

   You'll have to try working with a *matrix* of Laplace transforms: just do what you think you should and it'll work out…