

Math 3D Differential Equations Homework Questions 6

1. Find the convolution $f(t) * g(t)$

(a) $f(t) = t \quad g(t) = e^{at}$

(b) $f(t) = e^{at} \quad g(t) = e^{bt} \quad (a \neq b)$. What happens if $a = b$?

2. Apply the convolution theorem to find the inverse Laplace transforms

(a) $F(s) = \frac{1}{s(s^2 + 4)}$

(b) $F(s) = \frac{1}{s(s^2 + 4s + 5)}$

3. Use the convolution method to solve the initial value problem

$$x'' + 4x' - 5x = e^{-t^2}, \quad x(0) = 1, \quad x'(0) = -2$$

(You will have to leave your answer as an integral)

4. Solve the initial value problems using Laplace transforms: $\delta(t)$ is the Dirac delta function.

(a) $x'' + 2x' + x = t + \delta(t), \quad x(0) = 0, \quad x'(0) = 1$

(b) $x'' + 2x' + x = \delta(t) - \delta(t - 2), \quad x(0) = 2 = x'(0)$

(c) (Hard!) $x'' + 9x = \delta(t - 3\pi) + \cos 3t, \quad x(0) = 0, \quad x'(0) = 0$

5. (a) Express the solution to the initial value problems as an integral:

i. $x'' + 4x = f(t), \quad x(0) = 0 = x'(0)$

ii. $x'' + 4x' + 8x = f(t), \quad x(0) = 0 = x'(0)$

(b) For the problem in part (a)(i), explicitly evaluate the integral when $f(t) = 1 - u(t - 1)$.

6. (Hard) A thin beam is simply supported at $x = 0$ and $x = 2$ meters.

(a) Suppose that the beam has a uniformly distributed load of 1 Newton per meter so that the shape of the beam (for $0 \leq x \leq 2$) satisfies the differential equation

$$\frac{d^4 y}{dx^4} = -1, \quad y(0) = y''(0) = y(2) = y''(2) = 0$$

Find the maximum deflection of the beam $y(1)$.

(b) Now suppose that the beam is loaded such that the entire weight of 2 Newtons is placed in the center of the beam. The differential equation is now

$$\frac{d^4 y}{dx^4} = -w \delta_1(x), \quad y(0) = y''(0) = y(2) = y''(2) = 0$$

Again compute the maximum deflection $y(1)$.

(Let $Y(s) = \mathcal{L}\{y(x)\}$, and solve for $y'(0)$ and $y'''(0)$ after computing $y(x)$)

7. (a) Use the power series method to solve the equation $y' = -4x^2 y$ (compare your answer with an easier method).

(b) Use the power series method to find the degree five Maclaurin polynomial of the solution to the equation $y' = (1 - 4x^2)y$. Compare with the solution obtained using separation of variables.

(Lots more power series problems/solutions are in the textbook)