Math 3D: Pre-requisite Quiz

The questions on this quiz are to help remind you of some of the pre-requisites for the course (primarily 2B + 3A). If you are struggling, ask questions now and review the relevant material from previous classes. This won’t be collected or graded, it is purely meant as a self-test.

1. Evaluate the integral
   \[ \int_0^\pi e^{2x} \sin x \, dx \]

2. (a) Calculate the determinant of the matrix
   \[
   A = \begin{pmatrix}
   7 & 2 & 3 \\
   3 & -6 & 15 \\
   1 & 1 & -1
   \end{pmatrix}
   \]

   (b) Hence or otherwise, deduce the number of solutions to the system \( Ax = 0 \). (Do not find the solutions)

3. Let \( B \) be the matrix
   \[
   B = \begin{pmatrix}
   8 & 3 \\
   -18 & -7
   \end{pmatrix}
   \]

   (a) Calculate the eigenvalues and eigenvectors of the matrix \( B \)

   (b) Find a diagonal matrix \( D \) and an invertible matrix \( X \) such that \( B = XDX^{-1} \)

4. Consider the power series
   \[ y(x) = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^n \]

   (a) What is its interval of convergence?

   (b) Prove that
   \[
   \frac{dy}{dx} + 2xy = \frac{1}{2} + \frac{1}{3} x + \sum_{n=2}^{\infty} \frac{n^2 + n + 4}{n(n-1)(n+2)} x^n
   \]