

## Math 3D: Pre-requisite Quiz

The questions on this quiz are to help remind you of some of the pre-requisites for the course (primarily 2B + 3A). If you are struggling, ask questions now and review the relevant material from previous classes. This won't be collected or graded, it is purely meant as a self-test.

1. Evaluate the integral

$$\int_0^{\pi} e^{2x} \sin x \, dx$$

2. (a) Calculate the determinant of the matrix

$$A = \begin{pmatrix} 7 & 2 & 3 \\ 3 & -6 & 15 \\ 1 & 1 & -1 \end{pmatrix}$$

- (b) Hence or otherwise, deduce the number of solutions to the system  $A\mathbf{x} = 0$ . (*Do not find the solutions*)

3. Let  $B$  be the matrix

$$B = \begin{pmatrix} 8 & 3 \\ -18 & -7 \end{pmatrix}$$

- (a) Calculate the eigenvalues and eigenvectors of the matrix  $B$   
(b) Find a diagonal matrix  $D$  and an invertible matrix  $X$  such that  $B = XDX^{-1}$

4. Consider the power series

$$y(x) = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^n$$

- (a) What is its interval of convergence?  
(b) Prove that

$$\frac{dy}{dx} + 2xy = \frac{1}{2} + \frac{1}{3}x + \sum_{n=2}^{\infty} \frac{n^2 + n + 4}{n(n-1)(n+2)} x^n$$