Math 3D: Pre-requisite Quiz

These questions won't be collected or graded. They are a self-test to help refresh some of the pre-requisites for the course (primarily 2B & 3A). If you are struggling with questions 1–3, ask questions and review the relevant material. Question 4(b) should be doable, though it's unlikely you've encountered such a problem before.

1. Evaluate the integral

$$\int_0^{\pi} e^{2x} \sin x \mathrm{d}x$$

2. (a) Calculate the determinant of the matrix

$$A = \begin{pmatrix} 7 & 2 & 3 \\ 3 & -6 & 15 \\ 1 & 1 & -1 \end{pmatrix}$$

   (b) Hence or otherwise, deduce the number of solutions to the system $A\mathbf{x} = 0$.

3. Let B be the matrix

$$B = \begin{pmatrix} 8 & 3 \\ -18 & -7 \end{pmatrix}$$

   (a) Calculate the eigenvalues and eigenvectors of the matrix B

   (b) Find a diagonal matrix D and an invertible matrix X such that $B = XDX^{-1}$

4. Consider the power series

$$y(x) = \sum_{n=1}^{\infty} \frac{1}{n(n+1)} x^n$$

   (a) What is its interval of convergence?

   (b) By differentiating term-by-term, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = \frac{1}{2} + \frac{1}{3}x + \sum_{n=2}^{\infty} \frac{n^2 + n + 4}{n(n-1)(n+2)} x^n$$