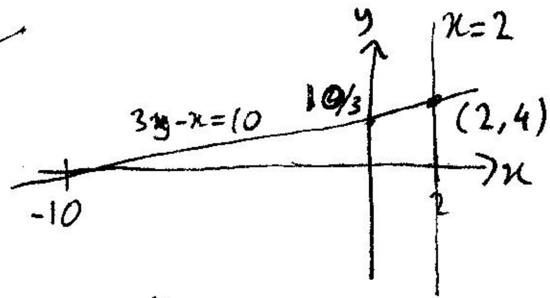
Math 4 - Homework 1 Answers

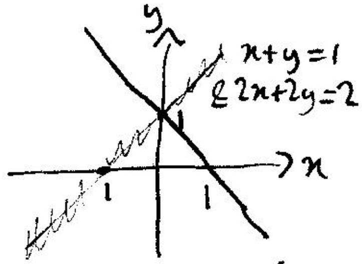
7.1

1/a/



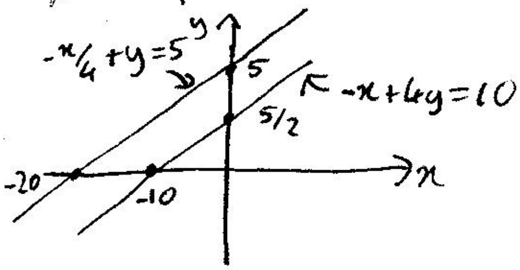
$x=2 \Rightarrow 3y = 10 + x = 12$
$\Rightarrow y = 4$

b/



∞ of sol$^{\underline{n}}$s - any points on the line

c/



no sol$^{\underline{n}}$s - parallel distinct lines.

2/a/ $\begin{cases} x+y=10 \\ 2x=4 \end{cases} \Rightarrow \begin{cases} x+y=10 \\ x=2 \end{cases} \Rightarrow \begin{cases} y=10-2=8 \\ x=2. \end{cases}$

c/ $\begin{cases} 2x+4y=2 \\ x+2y=1 \end{cases} \Rightarrow \begin{cases} x+2y=1 \\ x+2y=1 \end{cases} \Rightarrow \infty$ solutions (y = anything, $x = 1-2y$).

3/ $\begin{cases} 2x-y=10 \\ -cx+2y=5 \end{cases} \Rightarrow \begin{cases} 4x-2y=20 \\ -cx+2y=5 \end{cases} \Rightarrow \begin{cases} 2x-y=10 \\ (4-c)x=25 \end{cases}$

2nd eqn makes sense only if $c \neq 4$.

a/ $\therefore$ System has no sol$^{\underline{n}}$s if $c=4$ (get $0=25$ for eqn 2 above)

b/ System has one sol$^{\underline{n}}$ if $c \neq 4$

(namely $x = \dfrac{25}{4-c}$, $y = -10+2x = \dfrac{10c+10}{4-c}$)

6/ $\left.\begin{array}{l} q_1^s = q_2^d \\ q_2^s = q_2^d \end{array}\right\}$ supply = demand at equilibrium.

These give $\begin{cases} 2p_1 = 20 - p_1 + p_2 \\ -10 + 2p_2 = 40 - 2p_2 + p_1 \end{cases} \Rightarrow \begin{cases} 3p_1 - p_2 = 20 \\ p_1 - 4p_2 = -50 \end{cases}$

Substitute $\Rightarrow$ $3(4p_2 - 50) - p_2 = 20$

$\underset{11p_2 - 150}{\parallel}$

$\therefore 11p_2 = 170 \Rightarrow p_2 = \frac{170}{11} = 15\frac{5}{11}$

$\therefore p_1 = -50 + 4p_2 = -50 + \frac{680}{11} = \frac{130}{11} = 11\frac{9}{11}$

$\therefore q_1 = 2p_1 = \frac{260}{11} = 23\frac{7}{11}$

$q_2 = -10 + 2p_2 = -10 + \frac{340}{11} = \frac{230}{11} = 20\frac{10}{11}$

§7.2

1/b/ $\begin{cases} 4x - 2y + 2z = 6 \\ \quad\; -y + z = 1 \\ 2x \qquad\quad\;\; = 2 \end{cases} \Rightarrow \begin{cases} \quad -2y + 2z = 2 & (-2\times R_3 \text{ from } R_1) \\ \quad -y + z = 1 \\ x \qquad\quad\; = 1 & (\times \tfrac{1}{2}) \end{cases}$

$\Rightarrow \begin{cases} \quad -y + z = 1 & (-2\times R_2 \text{ from } R_1, \text{ get } 0 = 0) \\ x \qquad\; = 1 \end{cases}$

$\therefore \infty$ of solutions $(x, y, z) = (1, y, 1+y)$ where y is anything.

c/ $\begin{cases} -x_1 + 2x_2 - 3x_3 = 2 \\ \quad -2x_2 \qquad = 3 \\ 2x_1 - x_2 + x_3 = 4 \end{cases} \Rightarrow \begin{cases} -x_1 \qquad -3x_3 = 5 & (+R_2 \text{ to } R_1) \\ \qquad x_2 \qquad = -3/2 \\ 2x_1 \qquad +x_3 = 15/2 & (-\tfrac{1}{2}\times R_2 \text{ from } R_3) \end{cases}$

$\Rightarrow \begin{cases} -x_1 \qquad -3x_3 = 5 \\ \qquad x_2 \qquad = -3/2 \\ \qquad\qquad -5x_3 = \frac{35}{2} & (+2\times R_1 \text{ to } R_3) \end{cases} \Rightarrow x_3 = -7/2,\ x_2 = -3/2,$

$x_1 = -3x_3 - 5 = 11/2.$

2/ (Working required!)

a/ Consistent, sol^n $(x, y, z) = \left(\frac{1}{3}, \frac{8}{3}, \frac{-11}{3}\right)$

b/ Consistent since homogeneous. Only sol^n is $(0,0,0)$

c/ Consistent, sol^n $(2, -4, 1, 5)$

d/ (Harder) Depends on $a, b, c, \alpha, \beta, \gamma$.

$$\underset{\text{slope} = -\frac{a}{b}}{\underset{\uparrow}{ax+by=c}} \quad \& \quad \underset{\text{slope} = -\frac{\alpha}{\beta}}{\underset{\uparrow}{\alpha x+\beta y=\gamma}} \quad \text{are lines}$$

1 sol^n $\Leftrightarrow$ slopes distinct $\Leftrightarrow$ $\frac{a}{b} \neq \frac{\alpha}{\beta} \Leftrightarrow a\beta - b\alpha \neq 0$.

∞ sol^n $\Leftrightarrow$ lines identical $\Leftrightarrow$ $(a,b,c) = \lambda(\alpha,\beta,\gamma)$ for some constant λ.

0 sol^n $\Leftrightarrow$ lines // but not identical $\Leftrightarrow$ $(a,b) = \lambda(\alpha,\beta)$ and $c \neq \lambda\gamma$.

3/a/

$$\left(\begin{array}{cccc|c} 2 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 & 100 \\ 1 & 2 & 2 & 0 & 60 \\ -1 & 0 & 1 & -1 & -10 \end{array}\right) \to \begin{array}{c} -2R_3 \\ \\ \\ +R_3 \end{array} \left(\begin{array}{cccc|c} 0 & -5 & -4 & 0 & -120 \\ 0 & 1 & 1 & 2 & 100 \\ 1 & 2 & 2 & 0 & 60 \\ 0 & 2 & 3 & -1 & 50 \end{array}\right)$$

$$\rightsquigarrow \begin{array}{c} +5R_2 \\ \\ -2R_2 \\ -2R_2 \end{array} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 10 & 380 \\ 0 & 1 & 1 & 2 & 100 \\ 1 & 0 & 0 & -4 & -140 \\ 0 & 0 & 1 & -5 & -150 \end{array}\right) \rightsquigarrow \begin{array}{c} \\ -R_1 \\ \\ -R_1 \end{array} \left(\begin{array}{cccc|c} 0 & 0 & 1 & 10 & 380 \\ 0 & 1 & 0 & -8 & -280 \\ 1 & 0 & 0 & -4 & -140 \\ 0 & 0 & 0 & -15 & -530 \end{array}\right)$$

$$\rightsquigarrow \begin{array}{c} \\ \\ \\ \div -15 \end{array} \left(\begin{array}{cccc|c} 1 & 0 & 0 & -4 & -140 \\ 0 & 1 & 0 & -8 & -280 \\ 0 & 0 & 1 & 10 & 380 \\ 0 & 0 & 0 & 1 & 106/3 \end{array}\right) \rightsquigarrow \begin{array}{c} +4R_4 \\ +8R_4 \\ -10R_4 \\ \\ \end{array} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 0 & 8/3 \\ 0 & 0 & 1 & 0 & 80/3 \\ 0 & 0 & 0 & 1 & 106/3 \end{array}\right)$$

$$\therefore (x_1, x_2, x_3, x_4) = \left(\frac{4}{3}, \frac{8}{3}, \frac{80}{3}, \frac{106}{3}\right)$$

Note: There are other row operations you could use, but you will always end up with the same Reduced Row Echelon Form b/ $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 20 \\ -1 & 0 & 2 & 10 \end{array}\right) \rightsquigarrow \begin{array}{c} -R_2 \\ \\ +R_1 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -20 \\ 0 & 1 & 0 & 20 \\ 0 & 1 & 3 & 10 \end{array}\right) \rightsquigarrow \begin{array}{c} \\ \\ -R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -20 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 3 & -10 \end{array}\right)$

$\rightsquigarrow \begin{array}{c} \\ \\ \div 3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 1 & -20 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 1 & -10/3 \end{array}\right) \rightsquigarrow \begin{array}{c} -R_3 \\ \\ \\ \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & -50/3 \\ 0 & 1 & 0 & +20 \\ 0 & 0 & 1 & -10/3 \end{array}\right)$

$\therefore \; (x_1, \cancel{y_2} x_2, x_3) = \left(\frac{-50}{3}, 20, \frac{-10}{3}\right)$