Math 4 Homework 4

§11.1/3/ $\dfrac{\partial R}{\partial x_2} = \lim_{h\to 0} \dfrac{R(x_1, x_2+h) - R(x_1,x_2)}{h} = \lim_{h\to 0} \dfrac{p_1x_1 + p_2(x_2+h) - p_1x_1 - p_2x_2}{h}$

$= \lim_{h\to 0} \dfrac{p_2 h}{h} = p_2.$

As the number of sales of item 2 increases, the revenue increases p_2 times as much (multiply quantity by price)

6/ $\dfrac{\partial y}{\partial x_1} = \lim_{h\to 0} \dfrac{(x_1+h)x_2 - x_1x_2}{h} = \lim_{h\to 0} \dfrac{hx_2}{h} = x_2$

$\dfrac{\partial y}{\partial x_2} = \lim_{h\to 0} \dfrac{x_1(x_2+h) - x_1x_2}{h} = \lim_{h\to 0} \dfrac{x_1 h}{h} = x_1$

7/ $\dfrac{\partial y}{\partial x_1} = 5x_1^{-1/2}x_2^{1/3}x_3^{1/4}$, $\dfrac{\partial y}{\partial x_2} = \dfrac{10}{3}x_1^{1/2}x_2^{-2/3}x_3^{1/4}$, $\dfrac{\partial y}{\partial x_3} = \dfrac{5}{2}x_1^{1/2}x_2^{1/3}x_3^{-3/4}$.

9/ $\dfrac{\partial y}{\partial x_1} = 12\cdot(-2)\left[0.4x_1^{-1/2} + 0.6x_2^{-1/2}\right]^{-3}\cdot\left(0.4\cdot\left(\tfrac{-1}{2}\right)x_1^{-3/2}\right)$

$= \dfrac{24}{5\left[x_1^{1/2}(0.4x_1^{-1/2} + 0.6x_2^{-1/2})\right]^3} = \dfrac{24}{5\left(0.4 + 0.6\left(\frac{x_1}{x_2}\right)^{1/2}\right)^3}$

$\dfrac{\partial y}{\partial x_2} = 12\cdot(-2)\left[0.4x_1^{-1/2} + 0.6x_2^{-1/2}\right]^{-3}\left(0.6\cdot\left(\tfrac{-1}{2}\right)x_2^{-3/2}\right)$

$= \dfrac{36}{5\left[x_2^{1/2}\left[0.4x_1^{-1/2} + 0.6x_2^{-1/2}\right]\right]^3} = \dfrac{36}{5\left(0.4\left(\frac{x_2}{x_1}\right)^{1/2} + 0.6\right)^3}.$

§11.2/3/ $f(x_1,x_2) = x_1^3x_2^4$.

$\nabla f = \begin{pmatrix} 3x_1^2x_2^4 \\ 4x_1^3x_2^3 \end{pmatrix}$, $D_2 f = \begin{pmatrix} 6x_1x_2^4 & 12x_1^2x_2^3 \\ 12x_1^2x_2^3 & 12x_1^3x_2^2 \end{pmatrix}$

4/ $f(x_1,x_2,x_3) = x_1^2 x_2^4 x_3^5$

$$\nabla f = \begin{pmatrix} 2x_1 x_2^4 x_3^5 \\ 4x_1^2 x_2^3 x_3^5 \\ 5x_1^2 x_2^4 x_3^4 \end{pmatrix}, \quad \nabla_2 f = \begin{pmatrix} 2x_2^4 x_3^5 & 8x_1 x_2^3 x_3^5 & 10x_1 x_2^4 x_3^4 \\ 8x_1 x_2^3 x_3^5 & 12x_1^2 x_2^2 x_3^5 & 20x_1^2 x_2^3 x_3^4 \\ 10x_1 x_2^4 x_3^4 & 20x_1^2 x_2^3 x_3^4 & 20x_1^2 x_2^4 x_3^3 \end{pmatrix}$$

6/ $f(x_1,x_2,x_3) = a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2$

$$\nabla f = 2\begin{pmatrix} a_1 x_1 \\ a_2 x_2 \\ a_3 x_3 \end{pmatrix}, \quad \nabla_2 f = \begin{pmatrix} 2a_1 & 0 & 0 \\ 0 & 2a_2 & 0 \\ 0 & 0 & 2a_3 \end{pmatrix}$$

9/ $f(x_1,x_2,x_3) = x_1^2 e^{3x_2 + x_1 x_3} + 2x_2^3/x_1$,

$f_1 = 2x_1 e^{3x_2+x_1x_3} + x_1^2 x_3 e^{3x_2+x_1x_3} - 2x_2^3 x_1^{-2}$,

$f_2 = 3x_1^2 e^{3x_2+x_1x_3} + 6x_2^2/x_1$, $f_3 = x_1^3 e^{3x_2+x_1x_3}$,

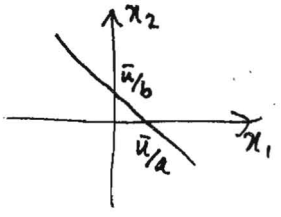
$f_{12} = 6x_1 e^{3x_2+x_1x_3} + 3x_1^2 x_3 e^{3x_2+x_1x_3} - 6x_2^2 x_1^{-2} = f_{21}$,

$f_{13} = 2x_1^2 e^{3x_2+x_1x_3} + x_1^2 e^{3x_2+x_1x_3} + x_1^3 x_3 e^{3x_2+x_1x_3} = f_{31}$.

§11.3/

2/a/ $u(x_1,x_2) = ax_1 + bx_2$, $\Rightarrow du = a\,dx_1 + b\,dx_2$.

b/



5/b/ $d\hat{u} = 2x_1 x_2^2 dx_1 + 2x_1^2 x_2 dx_2 \Rightarrow \dfrac{dx_2}{dx_1} = -\dfrac{2x_1 x_2^2}{2x_1^2 x_2} = -\dfrac{x_2}{x_1}$.

7/ $y = f(x_1, x_2) = \left[0.3x_1^{-3} + 0.7x_2^{-3}\right]^{-1/3}$

$f_1 = -\frac{1}{3}\left[0.3x_1^{-3}+0.7x_2^{-3}\right]^{-4/3}\left(0.3(-3)x_1^{-4}\right)$, $f_2 = -\frac{1}{3}\left[0.3x_1^{-3}+0.7x_2^{-3}\right]^{-4/3}\left(0.7(-3)x_2^{-4}\right)$

$\therefore MRTS = \frac{f_1}{f_2} = \frac{0.3x_1^{-4}}{0.7x_2^{-4}} = \frac{3x_2^4}{7x_1^4}$

8/ $dy = A\left(\frac{-1}{r}\right)\left(-r\delta x_1^{-r-1}\right)\left[\delta x_1^{-r}+(1-\delta)x_2^{-r}\right]^{-1/r-1} dx_1$

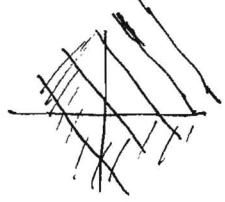
$+A\left(\frac{-1}{r}\right)\left(-r(\delta-1)x_2^{-r-1}\right)\left[\delta x_1^{-r}+(1-\delta)x_2^{-r}\right]^{-1/r-1} dx_2$

$\therefore MRTS = \frac{-r\delta x_1^{-r-1}}{-r(\delta-1)x_2^{-r-1}} = \frac{\delta x_2^{r+1}}{(\delta-1)x_1^{r+1}}$

Chap II Review

1/ $f(x_1,x_2) = Ax_1^{\alpha}x_2^{\beta}$. $f_1 = A\alpha x_1^{\alpha-1}x_2^{\beta}$, $f_2 = A\beta x_1^{\alpha}x_2^{\beta-1}$

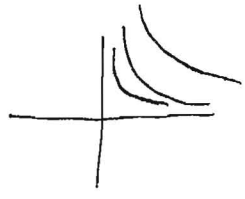
4/a/ $dy = dx_1 + dx_2$, slope $= \frac{dx_2}{dx_1} = -1$



only $x_1, x_2 \geq 0$

b/ $dy = \frac{1}{2}x_1^{-1/2}x_2^{1/3}dx_1 + \frac{1}{3}x_1^{1/2}x_2^{-2/3}dx_2$,

slope $\frac{dx_2}{dx_1} = \frac{-\frac{1}{2}x_1^{-1/2}x_2^{1/3}}{\frac{1}{3}x_1^{1/2}x_2^{-2/3}} = \frac{-3x_2}{2x_1}$



c/ $dy = 3x_1^2x_2^2dx_1 + 2x_1^3x_2dx_2$, slope $= \frac{dx_2}{dx_1} = \frac{-3x_1^2x_2^2}{2x_1^3x_2} = \frac{-3x_2}{2x_1}$