Math 4: Mathematics for Economists

Total 50 marks: marks per question are in brackets. You must show working for calculations, merely stating the answer will get you no marks

1. Calculate the determinant of the matrix (5)

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 3 & -6 & 15 \\ 1 & 1 & -1 \end{pmatrix}.$$

   Hence or otherwise, calculate the determinant of the matrix $\frac{1}{3}A$. (2)

2. Consider the following system of equations

$$\begin{cases} x + 2y + z = 3, \\ -2x + y - 4z = 3, \\ 3x - 2y \phantom{{}-4z} = 7. \end{cases}$$

   Find the Reduced Row Echelon form of the augmented matrix of the system and use it to calculate the solutions to the system. (8)

3. Let A, B be $n \times n$ matrices. Is it true that

$$(A - 2B)^2 = A^2 - 4AB + 4B^2?$$

   If not, what should the right hand side be? (4)

4. Use Cramer's rule to calculate x if (6)

$$\begin{cases} 7x + 8y + 6z + 9w = 1, \\ 3y + 3z + 3w = 0, \\ 2z + 2w = 0, \\ 4w = 0. \end{cases}$$

5. Suppose that the markets for tea, coffee and sugar are described by the supply and demand functions

$$\begin{aligned} D_t &= 90 - 3p_t + 2p_c - p_s, & S_t &= -6 + p_t, \\ D_c &= 88 + 2p_t - 3p_c - p_s, & S_c &= -8 + p_c, \\ D_s &= 36 + p_t + p_c - p_s, & S_s &= -12 + 2p_s, \end{aligned}$$

   where D_t, D_c, D_s are, respectively, the demanded quantities of tea, coffee and sugar, S_t, S_c, S_s the supplied quantities, and p_t, p_c, p_s the prices.

   (a) Show that the vector of equilibrium prices satisfies the matrix equation (5)

$$\begin{pmatrix} 4 & -2 & 1 \\ -2 & 4 & 1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} p_t \\ p_c \\ p_s \end{pmatrix} = \begin{pmatrix} 96 \\ 96 \\ 48 \end{pmatrix}.$$

(b) Calculate the equilibrium prices and quantities of tea, coffee and sugar.   (5)

6. Consider the matrix $A = \begin{pmatrix} -2 & 2 \\ 2 & 1 \end{pmatrix}$.

   (a) Calculate the eigenvalues and eigenvectors of A.   (5)

   (b) Find a matrix X and a diagonal matrix D such that $A = XDX^{-1}$.   (3)

7. Consider the quadratic form $g(\mathbf{x}) = 3x^2 + 2y^2 + 2z^2 + 2xy - 4xz$. Show that g is positive definite.   (7)