Math 4 Midterm Answers

1/ $|A| = \begin{vmatrix} 2 & 3 \\ -6 & 15 \end{vmatrix} - \begin{vmatrix} 4 & 3 \\ 3 & 15 \end{vmatrix} - \begin{vmatrix} 4 & 2 \\ 3 & -6 \end{vmatrix} = 48 - 51 + 30 = 27$

2/ $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ -2 & 1 & -4 & 3 \\ 3 & -2 & 0 & 7 \end{array}\right) \rightsquigarrow \begin{matrix} \\ +2R_1 \\ -3R_1 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 5 & -2 & 9 \\ 0 & -8 & -3 & -2 \end{array}\right) \rightsquigarrow \begin{matrix} \\ \\ +2R_2 \end{matrix} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 5 & -2 & 9 \\ 0 & -18 & 1 & -20 \end{array}\right)$

$\rightsquigarrow \begin{matrix} -R_3 \\ +2R_3 \\ \\ \end{matrix} \left(\begin{array}{ccc|c} 1 & 20 & 0 & 23 \\ 0 & -31 & 0 & -31 \\ 0 & -18 & 1 & -20 \end{array}\right) \rightsquigarrow \div 31 \left(\begin{array}{ccc|c} 1 & 20 & 0 & 23 \\ 0 & 1 & 0 & 1 \\ 0 & -18 & 1 & -20 \end{array}\right)$

$\rightsquigarrow \begin{matrix} -20R_2 \\ \\ +18R_2 \end{matrix} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right) \Rightarrow x = 3, y = 1, z = -2.$

3/ $(A - 2B)^2 = (A - 2B)(A - 2B) = A^2 - 2AB - 2BA + 4B^2.$

4/ $x = \dfrac{1}{\begin{vmatrix} 7 & 8 & 6 & 9 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 4 \end{vmatrix}} \begin{vmatrix} 1 & 8 & 6 & 9 \\ 0 & 3 & 3 & 3 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 4 \end{vmatrix} = \dfrac{24}{7 \times 24} = \dfrac{1}{7}.$

5/a, Applying $D_t = S_t$, $D_c = S_c$, $D_s = S_c$ we get.

$$\begin{cases} 40 - 3p_t + 2p_c - p_s = -6 + p_t \\ 88 + 2p_t - 3p_c - p_s = -8 + p_c \\ 36 + p_t + p_c - p_s = -12 + 2p_s \end{cases}$$

$$\Rightarrow \begin{cases} 4p_t - 2p_c + p_s = 46 \\ -2p_t + 4p_c + p_s = 96 \\ -p_t - p_c + 3p_s = 48 \end{cases} \Rightarrow \begin{pmatrix} 4 & -2 & 1 \\ -2 & 4 & 1 \\ -1 & -1 & 3 \end{pmatrix} \begin{pmatrix} p_t \\ p_c \\ p_s \end{pmatrix} = \begin{pmatrix} 46 \\ 96 \\ 48 \end{pmatrix}$$

b, Invert matrix: det $= 48 + 2 + 2 - 12 + 4 + 4 = 48$

$M_{11} = 13$, $M_{12} = -5$ $M_{13} = 6$

$M_{21} = -5$ $M_{22} = 13$ $M_{23} = -6$

$M_{31} = -6$ $M_{32} = 6$ $M_{33} = 12$

$$\therefore \begin{pmatrix} p_t \\ p_c \\ p_s \end{pmatrix} = \frac{1}{48}\begin{pmatrix} 13 & 5 & -6 \\ 5 & 13 & -6 \\ 6 & 6 & 12 \end{pmatrix}\begin{pmatrix} 96 \\ 96 \\ 48 \end{pmatrix} = \begin{pmatrix} 13 & 5 & -6 \\ 5 & 13 & -6 \\ 6 & 6 & 12 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 30 \\ 30 \\ 36 \end{pmatrix}$$

$$q_t = -6 + p_t = 24$$

$$q_c = -8 + p_c = 22$$

$$q_s = -12 + 2p_s = 60.$$

6/ a/ $|A - \lambda I| = \begin{vmatrix} -2-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = (\lambda - 1)(\lambda + 2) - 4 = \lambda^2 + \lambda - 6 = (\lambda+3)(\lambda-2)$

$\therefore$ e'values $\lambda_1 = 2$, $\lambda_2 = -3$

$\underline{\lambda_1 = 2}$ $\quad (A - \lambda_1 I)\underline{v}_1 = 0 \Rightarrow \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}\underline{v}_1 = 0 \Rightarrow \underline{v}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\underline{\lambda_2 = -3}$ $\quad (A - \lambda_2 I)\underline{v}_2 = 0 \Rightarrow \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}\underline{v}_2 = 0 \Rightarrow \underline{v}_2 = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

b/ $\quad A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}^{-1}.$

7/ $\underline{\text{Method 1}}$ $\quad g(\underline{x}) = (x\ y\ z)\begin{pmatrix} 3 & 1 & -2 \\ 1 & 2 & 0 \\ -2 & 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Leading principal minors

are 3, $\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 5$, $\begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 0 \\ -2 & 0 & 2 \end{vmatrix} = 12 - 2 - 8 = 2$

All $> 0 \Rightarrow g$ is positive definite.

$\underline{\text{Method 2}}$ Observe that

$g(\underline{x}) = (x+y)^2 + 2(x-z)^2 + y^2$ which is > 0

unless $y = 0 = x = z$

$\therefore$ g is positive definite.