Math 8: Homework Questions 2

Submit questions 2, 3(a), 4, 5(a,c)

1. If a line passes through points (x_0, y_0) and (x_1, y_1) show that it has equation

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0)$$

2. Find the equations of all quadratic polynomial functions which pass through the points $(1, 3)$ and $(2, 4)$.

3. For the following quadratic functions, complete the square and use your answer to graph the function.

   (a) $f(x) = x^2 - 6x + 5$
   (b) $f(x) = -x^2 + x + 1$
   (c) $f(x) = -3x^2 + 8x + 5$

   For part (a), also find two intervals $(-\infty, k]$ and $[k, \infty)$ (same k!) on which f is invertible. For each interval, compute the inverse function f^{-1}.

4. Consider the frisbee/tree problem in the notes. If you throw the ball in such a way that the initial *vertical* speed of the ball is twice its *horizontal* speed, find how fast you have to throw the ball in order to hit the frisbee.

   (*Hint: If $y(x)$ is the trajectory, you need $y'(0) = 2$: why?*)

5. Factorize the following polynomials and thereby find their (real) roots. Explain your steps carefully.

   (a) $f(x) = x^3 + 2x^2 - 3x$
   (b) $f(x) = x^4 - 13x^2 + 36$
   (c) $f(x) = x^3 - 7x - 6$
   (d) $f(x) = x^6 - 2x^5 - x^4 - 4x^3 - 4x^2 - 4x - 6$

6. Let $f(x) = ax^3 + bx^2 + cx + d$ is a cubic polynomial. 'Complete the cube' by finding a linear substitution $x - k$ such that

$$f(x) = a(x - k)^3 + p(x - k) + q$$

   has no $(x - k)^2$ term (i.e. find k, p, q which satisfy the above).

7. Let $A = (a, a^2)$, $B = (b, b^2)$ be any point on the graph of the function $f(x) = x^2$.

   (a) Find the equation $y = mx + c$ for the line joining A, B and verify that the segment $\overline{AB}$ lies above the graph of the parabola (i.e. $f(x) = x^2$ is convex).
   (b) By appealing to the idea of completing the square, explain *without calculation* why every quadratic polynomial is either concave up or concave down.