Math 8: Homework Questions 3

Submit questions 1(b), 2, 3, 4(a), 5, & 6 on Canvas by Thursday 21st April

1. Use Newton–Raphson iteration to find a root of the given function to 4dp. (*Use a calculator, but explain what you are doing!*)

   (a) $f(x) = x^3 - 4$ (b) $f(x) = 2x^3 + x - 1$

2. Use the Newton–Raphson method to find a rational number approximation to $\sqrt[3]{2}$ in lowest terms $\frac{p}{q}$ where $10 < q < 100$.

3. Suppose you perform the Newton–Raphson iteration for the function $f(x) = x^2 - 2$ starting with some positive $x_0 > 0$.

   (a) If $x_n > 0$, show that $x_{n+1} - \sqrt{2} = \frac{1}{2x_n}(x_n - \sqrt{2})^2$.

   (b) Explain why $\left|x_n - \sqrt{2}\right| < \frac{1}{2^n}\left|x_0 - \sqrt{2}\right|$. Hence conclude that the sequence of iterates (x_n) converges to $\sqrt{2}$.

4. Let $f(x) = x^3 - 5x$.

   (a) What happens if you apply Newton–Raphson iteration to this function with initial condition $x_0 = 1$? Draw a picture to illustrate.

   (b) Investigate what happens for other values of x_0. Can you make any conjectures? Is is possible for x_0 to be *positive* and yet for $x_n \to -\sqrt{5}$? Can you make any sense of what happens if $1 < x_0 < \sqrt{\frac{5}{3}}$?

5. Ten children had their heights (in inches) measured at their first and second birthdays. The data was as follows.

1st birthday	28	28	29	29	29	30	30	32	32	33
2nd birthday	30	32	31	34	35	33	36	37	35	37

   Given this data, use a regression model to predict the 2-year height of a child who measures 32 inches at age 1. What is the coefficient of determination and what does it say about your confidence in your prediction?

   (*You can—and should!—do this by hand. The averages are integers, so everything is easy…*)

6. (a) Let a, b be given. Find the value of y which minimizes the sum of squares

   $$(y - a)^2 + (y - b)^2$$

   (b) For the data set $\{(t, y)\} = \{(1, 3), (2, 1), (2, 4)\}$, find the least-squares linear model for predicting y given t.
   (*Hint: think about part (a) if you don't want to compute*)