Math 8: Homework Questions 5

Submit answers to questions 1, 2, 4 & 5 on Canvas by Thursday 19th May

1. For the following data sets, try to spot a pattern relating successive x and y-values. Use these to find a *function* $y = f(x)$ relating the two.

   (a)

x	2	4	6	8
y	-1	2	7	14

   (b)

x	20	60	180	540
y	2	4	6	8

   (c)

x	0	1	2	3	4
y	80	120	180	270	405

2. Suppose a table of data values containing (x_0, y_0) has constant first differences in both variables

   $$\Delta x = x_{n+1} - x_n = a, \quad \Delta y = b$$

   Find the equation of the linear function $y = f(x)$ through the data.

3. Suppose data has constant x-differences $\Delta x = a$ and constant second y-differences b. Explain how to find the formula for the *quadratic* function $y = f(x)$ passing through the data.
   (*Don't compute the full expression unless you're feeling masochistic!*)

4. How does our analysis of exponential functions change if we add a constant to the model? That is, how might you recognize a sequence arising from a function $f(x) = ba^x + c$?

5. You are given the following data for the population of Long Beach CA, every 10 years.

Year	Years since 1900	Population
1900	0	2,252
1910	10	17,809
1920	20	55,593
1930	30	142,032
1940	40	164,271
1950	50	250,767
1960	60	334,168
1970	70	358,879
1980	80	361,498
1990	90	429,433
2000	100	461,522
2010	110	462,257

   Use a spreadsheet or matlab to find linear, quadratic, exponential and logarithmic regression models for this data. Which of these seems to best fit the data and which would you trust to best predict the population in 2020? Look up the population of Long Beach in 2020; does it confirm your suspicions? What is going on?

6. Suppose $f(x)$ is a twice differentiable function and $h > 0$ is constant. Use the mean value theorem from calculus to explain the following.

   (a) First differences $f(x+h) - f(x)$ are proportional to $f'(\xi)$ for some $\xi \in (x, x+h)$.

   (b) Second differences satisfy $\big(f(x+2h) - f(x+h)\big) - \big(f(x+h) - f(x)\big) = f''(\xi)h\alpha$ for some ξ between x and $x+h$ and some α. Why is it unlikely that α is constant?