Math 8: Homework Questions 6

Submit answers to questions 1, 2, 3, 5 & 7 on Canvas by Thursday 26$^{\text{th}}$ May

1. Find the focus and directrix of the following parabolas and sketch them.

    (a) $y = 4x^2 - 24x + 38$
    (b) $y = -2x^2 - 12x$
    (c) $x + y^2 = 1$

2. For each of the following, complete the square to find the foci and sketch the curve.

    (a) $x^2 + 4y^2 - 2x + 16y = 17$
    (b) $2x^2 - 3y^2 + 8x - 18y = 5$

3. The equation $xy = 1$ is a hyperbola. By making the substitution $x = \frac{1}{\sqrt{2}}(u - v)$, $y = \frac{1}{\sqrt{2}}(u + v)$ show that this is indeed the case. Find the foci and directrices of this hyperbola.
   (*This change of co-ordinates amounts to rotating the plane by 45°*)

4. Suppose ℓ is a line parallel to the semi-major axis of an ellipse. If ℓ intersects the ellipse at two points, prove that the distance from each intersection to the nearest focus is the same.

5. Recall that a hyperbola is defined as the set of points P such that the difference of the distances

$$|FP| - |GP| = \pm 2a, \qquad a < \frac{1}{2}|FG|$$

   from two fixed points (the foci) is constant. Derive the canonical form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. In these co-ordinates, what are the equations of the directrices?

6. Suppose F is a point not on a line d. If $0 < e < 1$, use algebra to verify that the set of points P satisfying

$$|FP| = e|Pd|$$

   really does describe half an ellipse.

7. Let p be a constant and consider the intersection of the cone $z^2 = x^2 + y^2$ with the plane $z = py + 1$. Describe how the type of conic obtained relates to the value p.