MATH 150 HOMEWORK 1

DUE: Wednesday, Oct 11
Student Name/ID # (Include all students in the group):

IMPORTANT INSTRUCTION: It is crucial that you clearly explain how you arrive at your conclusions. The point of the homework is to practice understanding of the material and the ability to express your understanding.

1. (**5 pts**) Determine whether the (tauto)logical implication really holds. In the affirmative case provide an explanation (if you use truth table to prove your assertion, try to give an argument using as few lines as possible). In the negative case give an example of an evaluation which guarantees $\not\models$.

(a) (**2 pts**) $\emptyset \models (\neg A \to (\neg B \to \neg C)) \to (C \to (B \to A))$.

(b) (**2 pts**) $\{\phi, \neg\phi\} \models \psi \wedge \neg\psi$ (here ϕ and ψ are arbitrary formulas).

(c) (**1 pt**) $\{\neg A \to A\} \models A$.

2. (**5 pts**)In this exercise we abandon our conventions about omitting parentheses, so we will write:

- the outermost parentheses to each formula, and
- the parentheses around the negation, i.e. we will write $(\neg\psi)$ in place of the shorter $\neg\psi$.

For each formula ϕ denote:

- $s(\phi)$ = the total number of symbols in ϕ, and
- $b(\phi)$ = the total number of occurrences of binary connectives (that is occurrences of $\wedge, \vee, \to, \leftrightarrow$) in ϕ.

So for instance if

$$\phi : (\neg((A \to B) \to (\neg C)))$$

then $s(\phi) = 15$ and $b(\phi) = 2$. Prove by induction on the complexity of formulas that for every formula ϕ we have

$$s(\phi) \geq 3b(\phi).$$

3. (**5 pts**) Prove or refute each of the following assertions (same guidelines as in Problem 1):

(a) (**2 pts**) α is tautological equivalent to β iff $\models \alpha \leftrightarrow \beta$.

(b) (**2 pts**) $\models (((A \wedge B) \to C) \leftrightarrow (A \to (B \to C)))$.

(c) (**1 pt**) If either $\Sigma \models \alpha$ or $\Sigma \models \beta$, then $\Sigma \models \alpha \vee \beta$.