MATH 150 HOMEWORK 4
Due: Nov 8

IMPORTANT INSTRUCTION: It is crucial that you clearly explain how you arrive at your conclusions.

1. (**5pts**) For each of the following structures find a language for the respective structure and describe the interpretations of symbols of your language. Recall that for each function and relational symbol you need to specify the number of arguments.

(a) **(1pt)** The domain of the structure is the set of all integers $\mathbb{Z}$. There are two specified objects in the domain, namely 0 and 1. The structure has usual operations of addition and multiplication. Additionally, there are the following functions: exponentiation $n \mapsto 2n$, the function which to each two integers assigns their distance, and a function that to each tuple of integers of length five assigns the largest element.

(b) **(2pts)** The domain of the structure is the set of all real numbers R. For each integer $n > 0$ the structure has the function that to each finite tuple of real numbers of length n assigns the mean value. Additionally, for each integer $n > 0$ the structure has the relation that for each finite tuple of real numbers of length n tells whether the distances between the adjacent numbers are all the same.

(c) **(2pts)** The domain of the structure is the set of all lines in the Euclidean plane. The structure has the relation "line l is parallel with line l'". Additionally, the structure has relations that for each triple of lines carry the information about how many lines in the triple are parallel – make this precise.

2. (**5 pts**) Given is a structure $\mathfrak{M}$ characterized as follows. The domain M of the structure $\mathfrak{M}$ is the set of all people. The language has one constant symbol p, two unary function symbols F and G and one binary function symbol C. The interpretation of the constant symbol p is:

$$p^{\mathfrak{M}} = \text{the president of the people in } M.$$

The interpretations $F^{\mathfrak{M}}, G^{\mathfrak{M}}$, and $C^{\mathfrak{M}}$ of these symbols are two unary functions

$$F^{\mathfrak{M}} : M \to M \text{ and } G^{\mathfrak{M}} : M \to M$$

and a binary function

$$C^{\mathfrak{M}} : M \times M \to M$$

with the following interpretations: For each $m \in M$,

$$\begin{aligned} F^{\mathfrak{M}}(m) &= \text{the mother of } m, \\ G^{\mathfrak{M}}(m) &= \text{ the father of } m, \end{aligned}$$

and for each pair $(m, n) \in M \times M$, $C^{\mathfrak{M}}(m, n) =$

- the oldest child of the couple (m, n) if (m, n) has a child;
- the oldest child amongst all children of m, n if the couple (m, n) doesn't have a child but at least one of m, n does have a child;
- the president if m, n do not have any children or (m, n) is not a couple.

For each of the following situations write down the term describing the following persons.

(a) **(1pt)** The great-grandmother of u from the father's and grandfather's side.

(b) **(1pt)** The grandfather from the mother's side of the grandmother from the mother's side of u.

(c) **(1pt)** The oldest sibling of u, granting that u has a sibling.

(d) **(1pt)** The oldest uncle/aunt of u from mother's side, granting u has one.

(e) **(1pt)** The husband of the president's daughter, granting that the president has only one child, this child is a daughter, and she has a child with her husband.

3. (**5pt**) Let $\mathcal{L} = \{\dot{0}, \dot{S}, \dot{+}, \dot{\times}, \dot{<}\}$ be the language of number theory introduced in class. Express each of the following statements as a formula (or sentence) in $\mathcal{L}$.

(a) **(1pt)** "Numbers u, v are consecutive primes".

(b) **(1pt)** "Numbers u, v are relative primes".

(c) **(1pt)** "w is the greatest common divisor of u, v"

(d) **(1pt)** "Numbers u, v, w, z constitute an arithmetic sequence"

(e) **(1pt)** "There are infinitely many primes"

Recall that numbers a, b are relative primes if they have only one common divisor, namely number 1. Also, d is the greatest common divisor of a, b iff d is a common divisor of a, b and every common divisor of a, b divides d. Also recall that an increasing sequence of numbers $(a_1, a_2, \ldots, a_k)$ is arithmetic if the distances between all consecutive numbers in the sequence are all equal.