Nam Trang
Math 280 Assignment #4 due Monday, November 9, 2015

**Problem 1.** Suppose $\kappa$ is a regular uncountable cardinal. Suppose $(X_\alpha : \alpha < \kappa)$ and $(Y_\alpha : \alpha < \kappa)$ are enumerations of the same collection of subsets of $\kappa$. Show that $\triangle_{\alpha<\kappa} X_\alpha = \triangle_{\alpha<\kappa} Y_\alpha$ (mod $NS_\kappa$).

**Problem 2.** Suppose $\kappa$ is a regular, uncountable cardinal and $\kappa \subseteq A$. Let $\wp_\kappa(A) = \{x \subset A \mid |x| < \kappa\}$. A $\kappa$-complete filter $\mathcal{F}$ on $\wp_\kappa(A)$ is normal if for every $a \in A$, $\{x \in \wp_\kappa(A) \mid a \in x\} \in \mathcal{F}$ and $\mathcal{F}$ is closed under diagonal intersection (i.e. if $(A_a : a \in A)$ are sets in $\mathcal{F}$ then $\triangle_{a \in A} A_a \in \mathcal{F}$, where $\triangle_{a\in A} A_a = \{x \in \wp_\kappa(A) \mid x \in \bigcap_{a \in x} A_a\}$). A set $X \subseteq \wp_\kappa(A)$ is $\mathcal{F}$-positive if its complement is not in $\mathcal{F}$. Show that if $g$ is a function on an $\mathcal{F}$-positve set such that $g(x) \in [x]^{<\omega}$ for all $x$, then $g$ is constant on an $\mathcal{F}$-positive set. (Hint: you should try to assume first the case $g(x) \in x$ for all $x$.)

**Problem 3.** Same hypothesis as in the previous problem. Show that if $F$ is a normal $\kappa$-complete filter on $\wp_\kappa(A)$ then $F$ contains all closed unbounded sets.

**Problem 4.** Show that:

(a) If $\aleph_\omega < 2^{\aleph_0}$ then $\aleph_\omega^{\aleph_0} = 2^{\aleph_0}$.

(b) If $2^{\aleph_1} = \aleph_2$ and $\aleph_\omega^{\aleph_0} > \aleph_{\omega_1}$, then $\aleph_{\omega_1}^{\aleph_1} = \aleph_\omega^{\aleph_0}$.

(c) If $\kappa$ is a singular cardinal such that $2^{\mathrm{cof}(\kappa)} < \kappa \leq \lambda^{\mathrm{cof}(\kappa)}$ for some $\lambda < \kappa$, then $\gimel(\kappa) = \gimel(\lambda)$ for the least $\lambda$ such that $\kappa \leq \lambda^{\mathrm{cof}(\kappa)}$.

**Problem 5.** Suppose $\kappa$ is singular cardinal. Show that there is no $\kappa$-complete, non-principal filter on $\kappa$.

**Problem 6.** Show that if $\mathcal{F}$ is a non-principal, normal, uniform filter on $\kappa$, a regular, uncountable cardinal. Then $\mathcal{F}$ is $\kappa$-complete and contains all club sets in $\kappa$. Here $\mathcal{F}$ is uniform if it contains all sets of the form $(\alpha, \kappa)$ for $\alpha < \kappa$ and normal if it is closed under diagonal intersections.