Nam Trang
Math 280 Assignment #5 due Friday, November 20, 2015

**Problem 1.** Prove that the club filter on $\wp_\kappa(\lambda)$, where $\kappa$ is a regular, uncountable cardinal and $\lambda \geq \kappa$ is a cardinal, is $\kappa$-complete and normal (in the sense defined in class).

**Problem 2.** Prove that a subset $A$ of $\omega_1$ is club in $\omega_1$ if and only if $A$ is club in $\wp_{\omega_1}(\omega_1)$.

**Problem 3.** Suppose $M, N \vDash \mathsf{ZFC}$ and are transitive. Suppose $j : M \to N$ is a nontrivial elementary embedding. Prove that:

(a) There is an ordinal $\alpha$ such that $j(\alpha) \neq \alpha$.

(b) For every ordinal $\alpha$, $j(\alpha) \geq \alpha$.

(c) Let $\alpha$ be the least such that $j(\alpha) \neq \alpha$, and hence $j(\alpha) > \alpha$. Show that $\alpha$ is a cardinal of $M$.

**Problem 4.** Let $j : V \to M$ be nontrivial elementary; here $M$ is a transitive class of $V$. Let $\kappa$ be the least ordinal $\alpha$ such that $j(\alpha) \neq \alpha$ (as followed from the previous problem). Let

$$U_j = \{A \subseteq \kappa \mid \kappa \in j(A)\}.$$

Show $U_j$ is a normal, $\kappa$-complete, nonprincipal ultrafilter on $\kappa$.

**Problem 5.** Show that if $\kappa$ is the least strongly inaccessible cardinal such that $\kappa$ is the $\kappa^{th}$ inaccessible. Show that $\kappa$ is not Mahlo.