MATH 13 SAMPLE MIDTERM EXAM

Instructions.

A. Please **show your work**, unless explicitly stated that showing your work is not required. Writing down a result without showing how you arrived at it will be considered unsatisfactory.

B. **All** electronic devices must be stored in a closed bag and **all wireless devices must be turned off**. In particular, it is not allowed to use **calculators of any kind**.

C. **Do problems you find easier first.** If you have spare time, check on correctness.

D. You are not allowed to use any books or notes.

E. Please present your UCI Id card when turning in your work.

Student Name:

Student Id Number:

Problem 1 (15 pts)	
Problem 2 (15 pts)	
Problem 3 (15 pts)	
Problem 4 (15 pts)	
Total (60 pts)	

1. (15 pts)

(a) (5 pts) Prove that $(A \cup B)/(A \cap B) = (A/B) \cup (B/A)$ by showing each side is contained in the other side. Illustrate what this equality is saying using a Venn diagram.

(b) (5 pts) Let $A = \{a\}$ and $B = \{\{a\}, b\}$. Compute $A \cap B, A \cup B, A \times B$, $\mathcal{P}(A) \times \mathcal{P}(B)$, and $\mathcal{P}(A \times B)$. You should list all elements of these sets.

(c) (5 pts) Fix two sets X, Y. Prove that $\mathcal{P}(X) \cup \mathcal{P}(Y) \subseteq \mathcal{P}(X \cup Y)$. Give a counterexample to show that we do not expect equality to hold.

2. (15 pts)

(a) (10 pts) Prove or disprove: If $f(x) : \mathbb{R} \to \mathbb{R}$ is an injective function, then $f(x)$ is not bounded above.

(b) (5 pts) Prove or disprove: Every function $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$ is onto.

3. (15 pts) A sequence $(x_n : n \in \mathbb{N})$ of real numbers is said to "converge to L" if for all $M \in \mathbb{R}$ and $M > 0$, there is an $N \in \mathbb{N}$ such that $n > N \Rightarrow |x_n - L| < M$. Here $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ is the set of natural numbers and $\mathbb{R}$ is the set of real numbers.

(a) (5 pts) Write the statement "$(x_n : n \in \mathbb{N})$ does not converge to 2" in mathematical form (with all quantifiers appropriately placed).

(b) (10 pts) Show that the sequence $(\frac{1}{(n+1)^2} : n \in \mathbb{N})$ converges to 0.

(c) (5 pts) Are the following statements logically equivalent? Explain your answer.

(a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}\ xy < y^2$.
(b) $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}\ xy < y^2$.

4. (15 pts)

(a) (5 pts) Prove or disprove: If n is not prime, then there exists an integer a which divides n and is in the interval $2 \leq a \leq \sqrt{n}$.

(b) (5 pts) Find an integer solution to the equation: $164x + 42y = 6$ if there is any. If there are no such solutions, say so.

(c) (5 pts) Using modular arithmetic to compute the reminder of $5^7 * 3^4 - 6^5$ when dividing this number by 4.