

Math 161 Modern Geometry Homework Questions 6

Due : Thursday, June 8, 2017

- (1) (a) Find the hyperbolic line in the Poincare disk model on which lie the points $(1/2, 0)$ and $(0, 1/4)$.
(b) Use your answer to find the hyperbolic distance between the points in part (a).
- (2) Let O be the origin and P be a point in the Poincare disk. Let r be the Euclidean distance between O and P . Show that the hyperbolic distance between O and P , $d = 2 \tanh^{-1}(r)$ or equivalently, $r = \tanh(d/2)$.
- (3) Show that if ℓ and m are limiting parallel lines, then they cannot have a common perpendicular.
- (4) Show that two hyperbolic lines cannot have more than one common perpendicular.
- (5) Let $PQ\Omega$ be an Omega-triangle. Prove that the sum of the angles $\angle PQ\Omega$ and $\angle QP\Omega$ is less than 180° .
- (6) Suppose that an Omega triangle is drawn with vertices at $O = (0, 0)$, $\Omega = (1, 0)$ and $P = (0, h)$ where $h > 0$. Prove that the hyperbolic line $P\Omega$ is an arc of a circle with equation $(x - 1)^2 + (y - k)^2 = k^2$ for some $k > 0$.
- (7) Prove that any hyperbolic line in the Poincare disk model of hyperbolic geometry is either a straight line, or an arc of a circle of the form $x^2 + y^2 + ax + by + 1 = 0$ with $a^2 + b^2 > 4$. Conversely, prove that any such arc is a hyperbolic line.