The point at ∞ & the extended complex plane: (29)

Define $\infty = \lim_{n\to\infty} z_n$ where $|z_n| \to \infty$.

Riemann sphere: $S^2 = \mathbb{C} \cup \{\infty\}$

Eg: $f: S^2 \to S^2$ $f(z) = \dfrac{1}{z-2}$ is bijective.

$$f^{-1}(z) = 2 + \frac{1}{z}$$

$f(2) = \infty$ $f(\infty) = 0$ $f^{-1}(\infty) = 2$

$f^{-1}(0) = \infty$.

Stereographic Projection:

Can visualize S^2 as the unit sphere with center $(0,0,0)$.

Identify $\mathbb{C}$ with the x-y-plane in $\mathbb{R}^3$.

Identify ∞ with $(0,0,1)$; let $\vec{a} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

If $z = x+iy$, identify z with the point $(x,y,0)$; let $\vec{b} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$.

Then

$$(e): \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + t \underbrace{\begin{bmatrix} x \\ y \\ -1 \end{bmatrix}}_{\vec{b}-\vec{a}}$$

is the vector equation of the line joining these points.

Stereographic Projection is the map that associates points $z = x+iy$ on the complex plane with points $P = (X, Y, Z)$ on the unit sphere

(and it associates $(0,0,1)$ with ∞)