Math 161 Modern Geometry Homework 2

Submit your answers to the following questions at the discussion on Tuesday Apr 25, 2017.

Guidelines:

- You should write legibly.
- You should write down your proof carefully and clearly state how you start the proof (direct proof, proof by contrapositive, by contradiction etc.) and write complete sentences.
- You can discuss homework with your classmates. But please clearly state who you work with on your homework. The cooperation stops at exchanging ideas. You must write the solutions on your own and not copy from your friend's homework.

1. Show that if two sides of a triangle are *not* congruent, then the angles opposite those sides are not congruent. *Hint: Use congruent triangles.*

2. Give a counterexample to the SSA (two congruent pairs of sides plus a congruent pair of angles but the angle is not between the sides) as a criterion for congruent triangles. Is AAS (two congruent pairs of angles plus a congruent pair of sides and the side is not in between the angles) a criterion for congruent triangles? Explain why/why not.

3. (a) In a triangle $\triangle ABC$, let points F and E be the midpoints of AB and AC respectively. Prove that FE is half the length of BC.

   (b) A *median* of a triangle is a segment from the midpoint of a side to the opposite vertex. Prove that the medians of a triangle meet in a single point. *This is the centroid.*

   (c) The medians of a triangle split the triangle into six sub-triangles. Prove that all six have the same area.

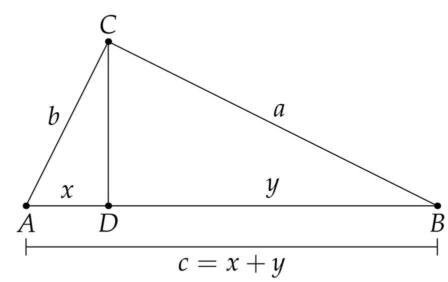
   (d) Prove that the centroid is exactly 2/3 of the distance along each median.

4. Let $\triangle ABC$ have a right-angle at C. Drop a perpendicular to AB from C.

   (a) Prove that you have *three* similar triangles.

   (b) Use these facts to prove Pythagoras' Theorem

$$a^2 + b^2 = c^2$$



5. Let the angle α be less than a straight edge. We say α is *acute* if it is less than a right-angle, and *obtuse* if it is greater. Its *supplementary angle* $\hat{\alpha}$ is the angle such that $\alpha + \hat{\alpha}$ is a straight edge.

   (a) Show that all triangles fall into exactly one of three cases:

      1. The triangle is right-angled.
      2. The triangle has three acute angles.
      3. The triangle has one obtuse and two acute angles.

   Your answer should not mention any *measure* (degrees, radians) of angles.

(b) In class we defined the sine and cosine of an acute angle α as ratios of lengths. Extend the definitions to right- and obtuse angles as follows:

$$\sin\alpha = \begin{cases} 1 & \text{if } \alpha \text{ is a right angle} \\ \sin\hat{\alpha} & \text{if } \alpha \text{ obtuse} \end{cases}$$

$$\cos\alpha = \begin{cases} 0 & \text{if } \alpha \text{ is a right angle} \\ -\cos\hat{\alpha} & \text{if } \alpha \text{ obtuse} \end{cases}$$

Use this definition to prove the *Sine Rule.* Suppose that a and b are the lengths of the edges of a triangle opposite angles α and β respectively. Then

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}.$$

Hint: Drop a perpendicular from C to AB at D and use the fact that you now have two right-triangles. The challenge is to make sure your answer is sufficiently general. Use part (a)...