# PROBLEM SET IV (DUE DEC. 7, 2018)

## 1. Problem 0

Rudin, Chapter 4, Exercise: 3, 4, 5, 6, 7, 8, 9, 14, 16, 18, 20, 21, 23, 24, 25(a).

## 2. Bonus problem 1

For a real number $x > 0$ let $\{x\}$ denote its fractional part, i.e., $\{x\} = x - [x]$ where $[x]$ denotes the largest integer smaller than $x$. Show that for any $x > 1$ we have

$$\sum_{k=1}^{n} \{kx\} \leq \frac{n}{2} x,$$

holds true for all $n \geq 1$.

## 3. Bonus problem 2

Let $f$ be a convex function such that the series $\sum_{k=1}^{\infty} kf(k)$ converges absolutely. Show that

$$\sum_{k=1}^{\infty} (-1)^{k-1} kf(k) \geq 0.$$

## 4. Bonus problem 3

Let $I \subset \mathbb{R}$ be an interval, and let $f : I \mapsto \mathbb{R}$ be a non-decreasing convex function. If $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are numbers in $I$ such that

$$x_1 \geq x_2 \geq \ldots \geq x_n,$$
$$y_1 \geq y_2 \geq \ldots \geq y_n,$$

and

$$x_1 + \ldots + x_k \geq y_1 + \ldots y_k, \quad \text{for all} \quad 1 \leq k \leq n$$

then

$$f(x_1) + \ldots + f(x_n) \geq f(y_1) + \ldots + f(y_n)$$