HOMEWORK V (DUE JUN. 7, 2019)

The purpose of the last homework will be to better understand differential forms.

Problem 1.

Consider 3-form in $\mathbb{R}^3$, $\omega = dx \wedge dy \wedge dz$. *Let* $\Phi : \mathbb{R}^3 \to \mathbb{R}^3$ *be a* C^1 *map*

$$\Phi(x, y, z) = (\varphi_1(x, y, z), \varphi_2(x, y, z), \varphi_3(x, y, z)).$$

We remind that by definition $\omega_\Phi = d\varphi_1 \wedge d\varphi_2 \wedge d\varphi_3$. *Show that*

$$\omega_\Phi = \det(\Phi')dx \wedge dy \wedge dz$$

where $\det(\Phi')$ *is the determinant of the matrix of the first derivatives of* Φ *differentiated with respect to the variables* x, y, z *in the order* (x, y, z).

Problem 2. *Let* $D = [0, 1] \times [0, 1]$. *Consider the two-form* $\omega = dx \wedge dy$ *in* $\mathbb{R}^2$. *Let us consider 2-surface* $\Phi(x, y) : D \to \mathbb{R}^2$, $\Phi(x, y) = (x, y)$, *and 2-surface* $\Psi : D \to \mathbb{R}^2$, $\Psi(x, y) = (y, x)$. *Show that* $\Phi(D) = \Psi(D) = D$, *and* $\int_\Phi \omega = -\int_\Psi \omega$.

Problem 3. *What is missing in the proof of Theorem 10.27 of Rudin?*

Problem 4. *Chapter 10, exercise 2; 7; 8; 9; 15; 16;*