PROBLEM SET I (DUE OCT. 26/2018)

1. Problem 1 (Proposition 1.18, Rudin)

The following statements are true in every ordered field

(a) If $x > 0$ then $-x < 0$, and vice versa.
(b) If $x > 0$ and $y < z$ then $xy < xz$.
(c) If $x < 0$ and $y < z$ then $xy > xz$.
(d) If $x \neq 0$ then $x^2 > 0$. In particular, $1 > 0$.
(e) If $0 < x < y$ then $0 < 1/y < 1/x$.

2. Problem 2

Let $\alpha \in \mathbb{R}$ be a cut, and let w be a positive rational number. Show that there exists an integer n such that $nw \in \alpha$ but $(n+1)w \notin \alpha$.

3. Problem 3

Let $1^* = \{p \in Q : p < 1\}$. Show that for any cut $\alpha \in R$, $\alpha > 0^*$, there exists a cut $\beta \in R$ such that $\alpha\beta = 1^*$.

4. The rest of the problems

Chapter 1, Exercise 4, 5, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18.

5. The last problem

Take any real number $t \leq 0$, and let $p \leq t$ and $q \leq t$. Let $a \geq 0$. Show that

$$p\sqrt{a^2 + (y+b)^2} + q\sqrt{a^2 + (y-b)^2} - 2ty \leq -a\left(\sqrt{p^2 - t^2} + \sqrt{q^2 - t^2}\right)$$

holds true for any real number $b \in R$ and any $y \geq 0$.