MIDTERM 205A, FALL 2018

Problem 1. *If $k \geq 2$ and $x \in \mathbb{R}^k$, prove that there exists $y \in \mathbb{R}^k$ such that $y \neq 0$ but $x \cdot y = 0$.*

Problem 2. *For real numbers $x, y \in \mathbb{R}$ define the function $d(x,y) = \frac{|x-y|}{1+|x-y|}$. Is it a metric? Explain your answer.*

Problem 3. *Give an example of an open cover of the* ***open*** *interval (segment) $(0,1)$ which has no finite subcover.*

Problem 4. *Let X be a metric space in which every infinite subset has a limit point. Prove that X is separable (separable means that it contains countable dense subset).*

Problem 5. *Let G be an open set in $\mathbb{R}$ which is not upper bounded. Does there exist a real number $b > 0$ such that the set $\{nb\}_{n \geq 1} \cap G$ is infinite?*