5

TEACHING MANUAL

Discovering Pentominoes

SPRING 2019

MEETING 5

MAY 7-8

Contents

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2019 UCI MATH CEO COMMUNITY EDUCATIONAL OUTREACH.
UNIVERSITY OF CALIFORNIA AT IRVINE

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Meeting 5
Discovering Pentominoes

- Tuesday 9:00 AM - 9:50 AM
  - Place: **UCI** NS2 1201 (Marco Forester comes)
- Tuesday 2:45 PM - 3:45 PM
  - Place: **SANTA ANA**: Carr Intermediate School
- Wednesday: 2:00 PM - 3:50 PM
  - Place 1: **UCI,** NS2 1201 : Lathrop comes
  - Place 2: **UCI,** PSCB 140 : Villa comes

<table>
<tr><td>Tuesday Morning
(50 minutes)
May 7: No meeting</td><td rowspan="2">Wednesday Afternoon
May 8
(80 minutes)

● Activity 1: Pentominos: 40 minutes
● Activity 2: GameTime 30 minutes
● Survey: 5 minutes</td></tr>
<tr><td>Tuesday Afternoon
(50 minutes)
May 7

● Activity 2: Game Time 45 minutes
● Survey: 5 minutes</td></tr>
</table>

ACTIVITY 1: DISCOVERING PENTOMINOES

Exploring spatial patterns and combinations through pentominoes

Time: 40-45 minutes

Pentominoes
(*pento = 5*)

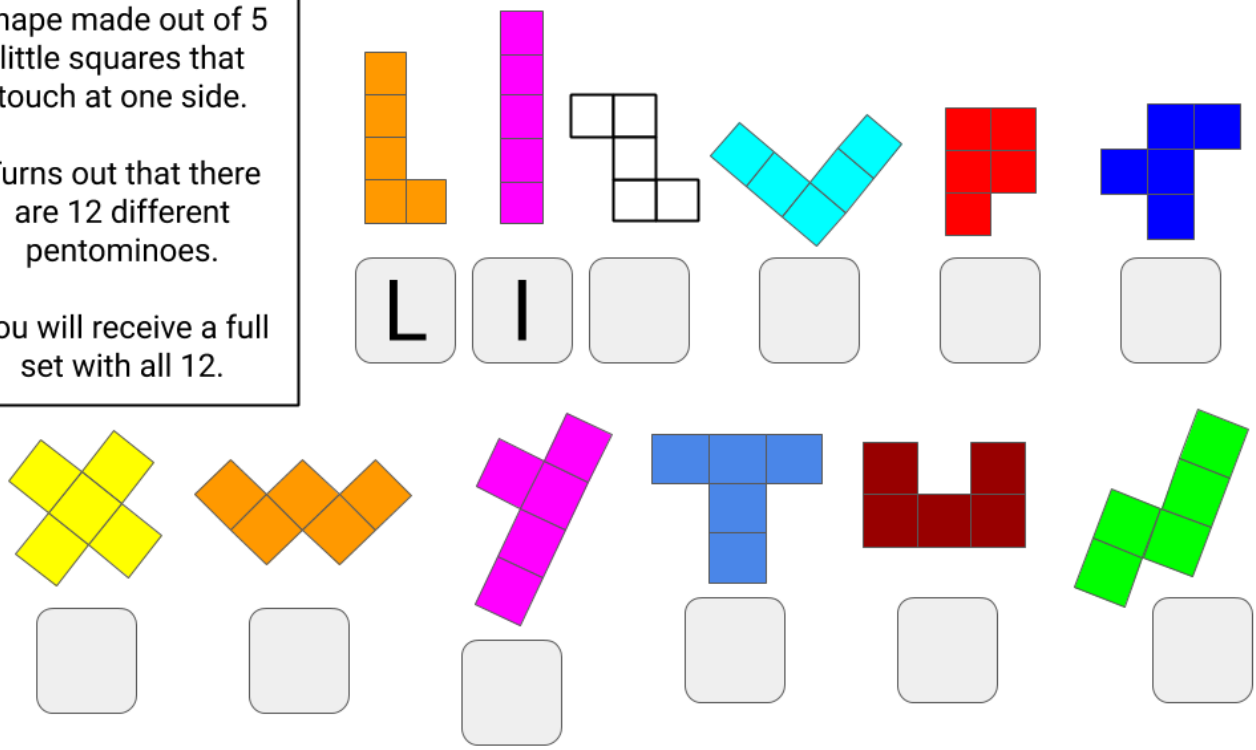
A pentomino is a shape made out of 5 little squares that touch at one side.

Turns out that there are 12 different pentominoes.

You will receive a full set with all 12.

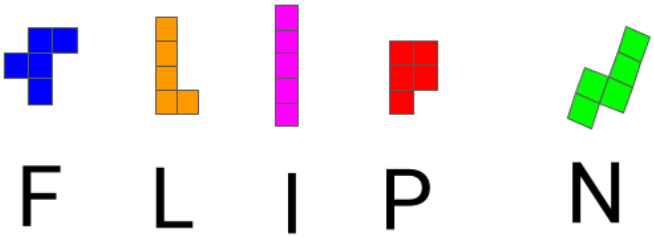
A) Labeling Pentominoes

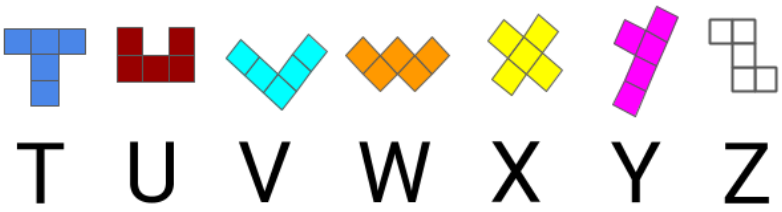
Label the 12 Pentominoes with letters…
How would you do so that you can remember them?



Do not look yet at the answers on the next page.

Here is the common way of labeling the pentominoes





All 12 pentominoes can be identified with 12 letters:

FLIPNTUVWXYZ

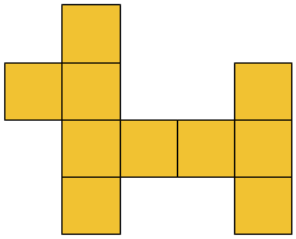
This is easy to remember!
"Flip N Tuvwxwy"

This is the convention of labeling the pentominoes that we will use in this activity

To help us all communicate, we will use this choice of letters to talk about all 12 pentominoes

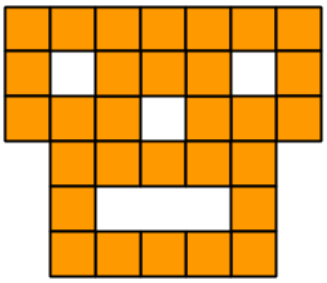
B) Pentomino shapes

i Use your pentominoes to make this **dog:**



How many pentominoes will you use? How can you know before you do it?

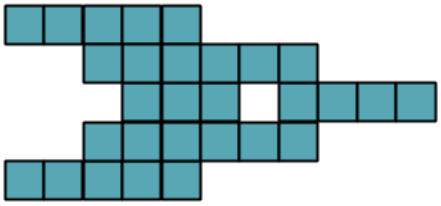
ii Use your pentominoes to make this **face of a monkey:**



How many pentominoes will you use? How can you know before you do it?

iii Can you make this **rocket?**

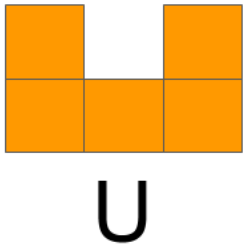
How do you know this?



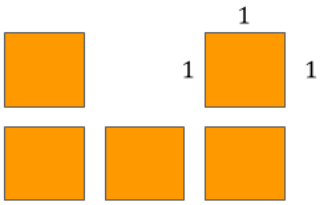
Remember to use only your tiles. So, you cannot use a pentomino more than once.
However, you can work in pairs to share insights of your work

C) Perimeters: the smallest and the largest

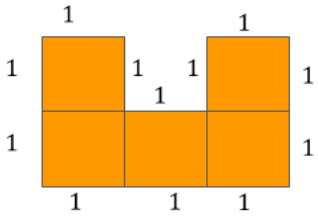
Each pentomino consists of 5 equal squares with side length 1



U



Remember that the perimeter of the pentomino is the sum of its boundary (border) side lengths



In this example, the perimeter is 12

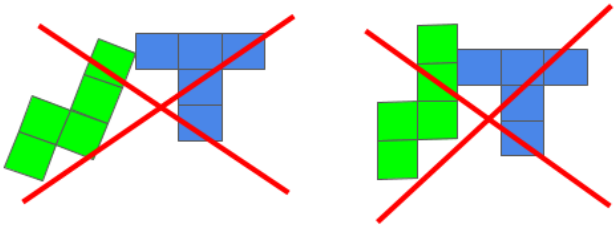
i Which pentomino has the smallest perimeter? Which strategy did you use?

ii Which pentomino has the largest perimeter? Which strategy did you use?

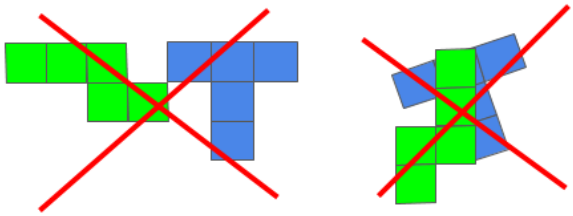
D) Double pentominoes

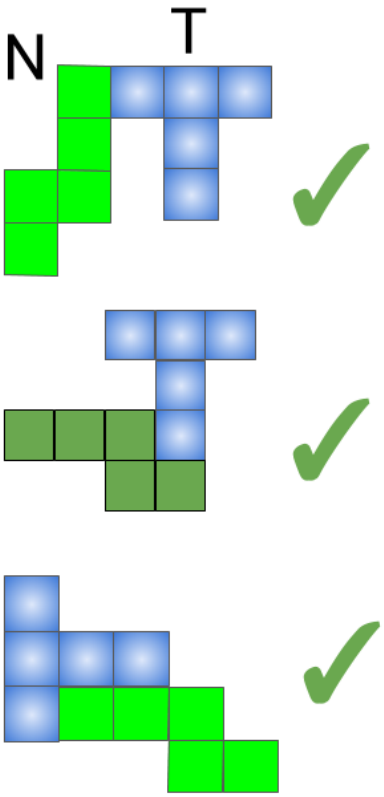
Put two pentominoes together on sides of their squares to make a figure. We call this a “double pentomino”.
Find the double pentomino with the smallest possible perimeter.
How many solutions can you find? What about the largest perimeter?

These pentominoes are not glued together, so they are not double pentominoes.



We need that the sides of the touching squares match perfectly (completely touch). Also, the little squares cannot overlap.

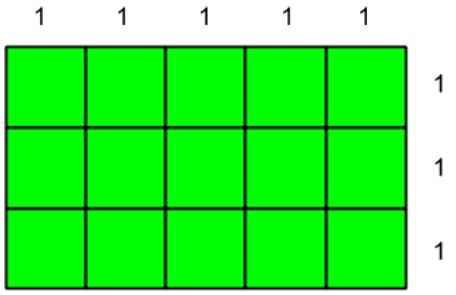




These shapes are all double pentominoes, because the pentominoes are well glued together by their square sides.

E) Making a 3x5 rectangle

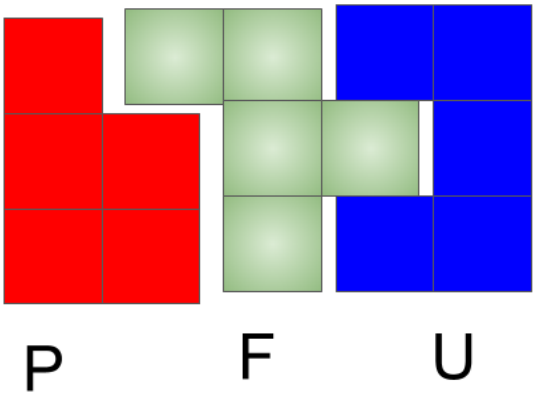
Put some of your pentominoes together to make 3x5 rectangle



How many different solutions can you find?

Record your solutions using the labeling of pentominoes.

One possible solution is: **PFU**

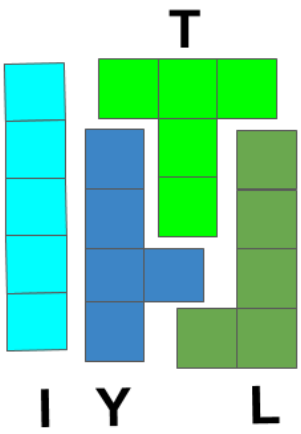


In this example presented below you would record your answer as

PFU

F) Making 4x5 rectangles

i Make at least five different 4x5 rectangles using pentominoes.
The construction of one such rectangle is pictured below.



Hint: use the following combination of pentominoes
FLPU, PUVZ, NTVY, IPUV, FPUY

Fact: there are 26 different solutions in total.

ii Here is a challenging question: do you think you can make 3x4 rectangles? Explain.

iii How many 2x10 rectangles can you make?

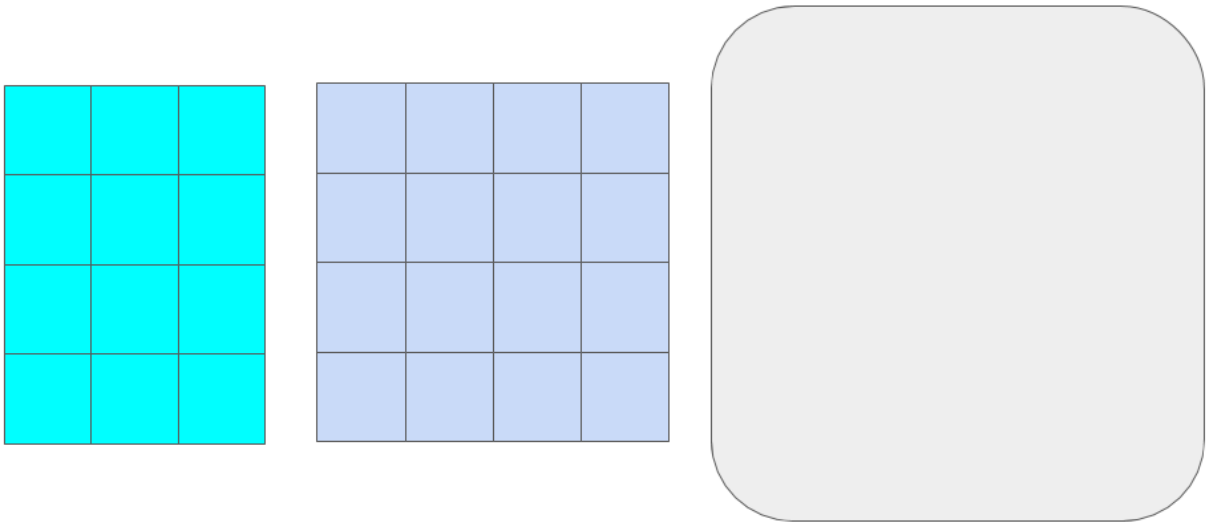
Hint: Can you use the **T** pentomino in your construction? Why or why not?

Put all those pentominoes aside which you cannot use for the construction of 2x10 rectangle.

Then, try to use the combination of the remaining ones. What happens?

G) 3x5 and 4x5 (at the same time)

Build two rectangular swimming pools simultaneously, using only your 12 pentominoes. One pool should have size 3x5, and the other should have size 4x5



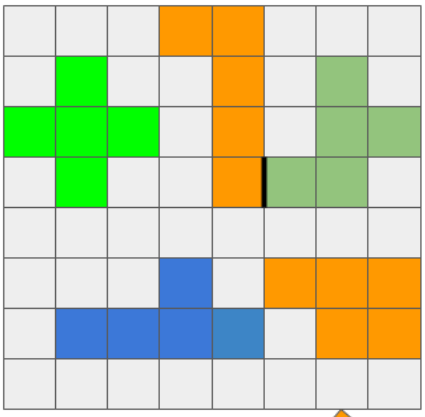
One solution can be found if you use the PUY and ILNV pentominoes.
Can you find other solutions?

H) Alice and Bob in Wonderland

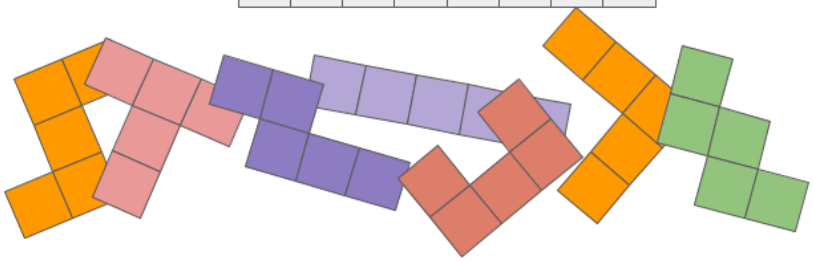
A Pentomino game

1. This game is for 2 players. Choose the second player.
2. Use the 8x8 board to play the game.
3. A. decide with each other who wants to make the first move.
   B. use only 12 pentominoes **for both players** (do not use 24 pentominoes)
   Put one pentomino on the 8x8 grid so that the squares of the pentomino coincide with the squares of the 8x8 table. Next, the second player puts her pentomino so that they do not overlap.
   The game continues until one player cannot make a move. That player loses.

Bob



Alice



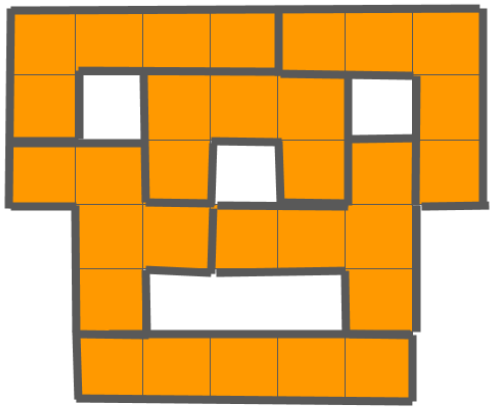
ACTIVITY 1: DISCOVERING PENTOMINOES	
Description	In this activity, students will explore pentominoes. They will solve perimeter and area problems and then play a spatial game using the 8x8 board provided. NOTES: ● There are several parts to this activity: A to H. Some are very fast, but some may take some time. You can offer some hints if you feel that some parts are taking a lot of time. You may skip some parts as well. Try that students get to the last part (H), in which they play a game.
Materials	● Student Workbook ● Pentomino tiles (One 12-piece set for each student) ● 8x8 grid (one per each pair of students, for the Pentomino Game)
Set-up	● You should let students first tackle the problem parts individually to give them some time to think on their own. After 1 or 2 minutes, you can let students explore it together.
Teaching tips	● Let kid explore the pentomino tiles for a few minutes before starting the math. ● Part A: This part helps students explore and get familiar with the tiles. Note that in this activity, students may come with their own labeling, but at the end of this part, ALL students should follow the labeling given in the solutions (both in the Teacher's manual and the Student Workbook). This is important so that everyone in the table can communicate with the same letter conventions. ● Part B: make sure that students can clearly explain why they cannot make the rocket. Also stress that, even if a shape has a number of squares that is a multiple of 5, it may be impossible to build it with pentominoes. So the condition "the shape has a number of squares that is a multiple of 5" is a necessary but not sufficient condition for building it with pentominoes. You may discuss this logic terminology, but it is not necessary to do so. Just anticipate these kinds of misconceptions from students. ● Part C: Some students may conclude (incorrectly) that all pentominoes have the same perimeter because they all have the same area. You may ask why they think that this is true, so that students can correct themselves. You may remind that area is a 2D property, whereas perimeter has to do with the borders and its 1D. ● Part D: In order to guide students a little bit, ask them whether they think that "longer" double pentominoes (as opposed to "square-like ones" --see solutions) have a larger or smaller perimeter and why. Students should realize that the more square like a double pentomino looks like, the smaller its perimeter seems to be (unless they are inner corners). Explore this intuition and have students check their conjectures.

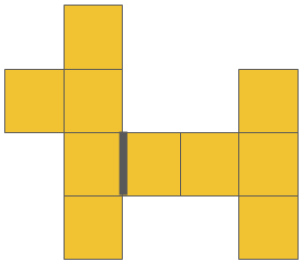
	● Part E: If students don't realize that the T pentomino can not be part of a 2x10 rectangle, don't tell them but ask them to try to build one. If they still don't realize, you could ask them to first draw a 2x10 rectangle in their whiteboards and then try to fill it out. This way, students will see that the T does not fit, no matter which way they orient it. ● Part H: During or after the game, ask students the following reflection questions (you can also make your own or have students ask among themselves): ○ Was it better to start the game, or it does not seem to matter? ○ Which was a good strategy to play the game? ○ What is the difference between playing close to the border or playing inside? Explain what you think.

Solutions (1: Pentominoes)

B)

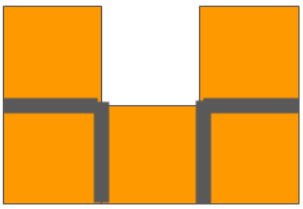
B) Solutions to making animals





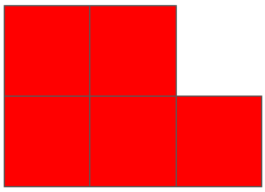
Solutions to C:
The minimal perimeter is 10.

Indeed, if the little squares of the pentominoes were not glued together, then they would all have the same perimeter because they all have the same number of squares (5), So it is enough to count how many glued sides we have in each pentomino to detect the one (or ones) with smallest perimeter. In this example the total number of glued sides is 4.



U

So the pentomino U has a perimeter of 20 minus 2x4 = 12. (We subtract 2x4, because there are 4 glued sides, and each glue side means that the perimeter goes down by 2 (since you are gluing 2 sides).

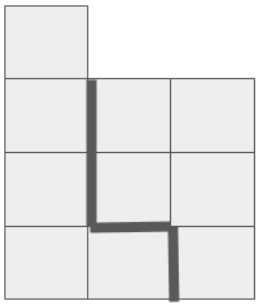


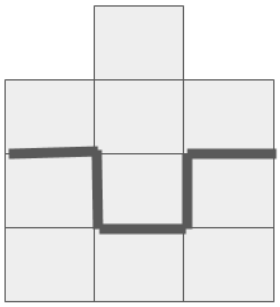
P

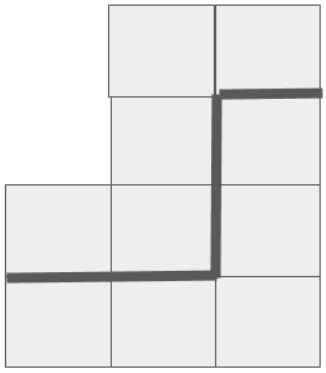
This pentomino has 5 glued sides. And only this pentomino has the smallest possible perimeter.

The largest perimeter is 12, and all pentominoes have largest perimeter except of P.

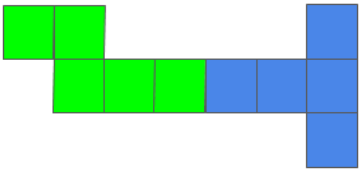
Solution to D):
The shortest perimeter of 2 glued pentominoes is 14. Here are 3 solutions to this problem:







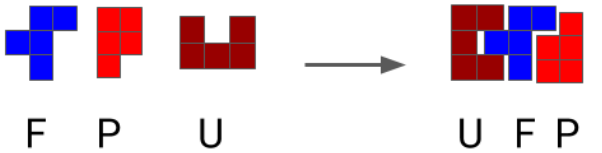
Any two pentominoes which touch each other at exactly one side (except of P) gives the maximal total perimeter 22

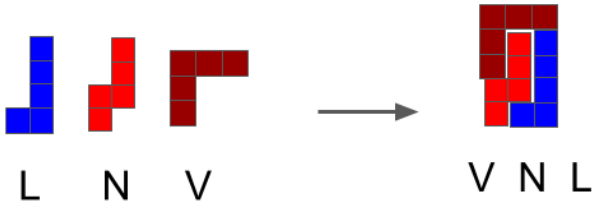


Note that each pentomino (except P) has perimeter 12. If we put them together to touch at one side, we get 24-2 = 22.

E) Solutions of 3x5 rectangles: there are 7 solutions in total:

FPU
LNV
LPV
LTY
NPU
PUV
PUY



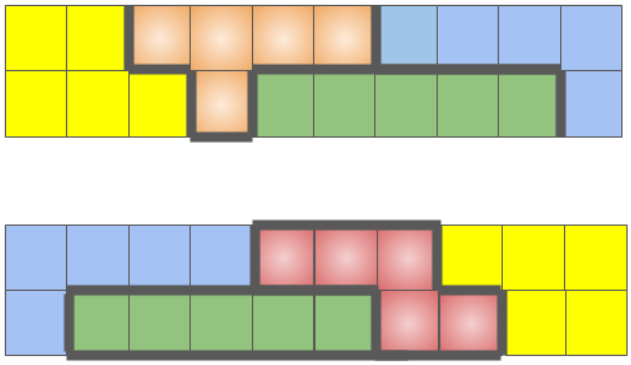


F) Solutions to 4x5 rectangles:

FIPU, FLPU, FLTY, FLUV, FLUY, FPUY, ILNV, ILPV, ILTY, INPU, IPUV, IPUY, LNPU, LNVZ, LPTV, LPTY, LPUY, LPVY, LPVZ, LPWY, LPYZ, LUVY, NPUY, NTVY, PTVW, PUVZ

You cannot make rectangle of size 3x4 because 12 is not divisible by 5, and each pentomino has an area of 5. So you would need to use more than two but less than 3 pentominoes, which is not possible.

You can make only two rectangles of size 2x10:



G) Solutions

There are 17 solutions in total.

Even if we know which pentominoes should be used for the solutions it is not easy to build the rectangles. So feel free to give students combinations of the pentominoes that they should use for the construction.

PUY-ILNV
PUY-LNVZ
FPU-ILNV
FPU-ILTY
FPU-LNVZ
FPU-NTVY
LNV-FIPU
LNV-FPUY
LNV-IPUY
NPU-FLTY
NPU-ILTY
PUV-FLTY
PUV-ILTY
LTY-FIPU
LTY-INPU
LTY-IPUV
LTY-PUVZ

ACTIVITY 2: GAME TIME

Using board games to think mathematically.

Time: 30 minutes

<table>
<tr><th colspan="2">ACTIVITY 2: GAME TIME</th></tr>
<tr><td>Description</td><td>During the quarter and especially in Wednesday's meetings, students will play different board games that will be available and talk about the experience in terms of the mathematics that they used and learned.</td></tr>
<tr><td>Materials</td><td>

Physical board games (each has a code for easy identification, and sometimes there are several copies of each game).
This time there will be just 1 game per table.
Board game rules are here:
https://drive.google.com/open?id=14MTdFPuZvo7flw2ch01dREZJ16n9e28s

</td></tr>
<tr><td>Set-up</td><td>
Wednesday: Each table picks one package from Q1 to Q18. Each package contains one or more copies of the same game.

In the following tables, you can see which package to pick (this may change, in which case our room leaders will let you know):

<table>
<tr><th colspan="3">Meeting 5: May 8, at NS II 1201 (Lathrop)</th></tr>
<tr><th>YOUR TABLE</th><th>PACKAGE</th><th>GAME(S)</th></tr>
<tr><td>1A</td><td>P3</td><td>Dara
This is a 2 player game. There is no game box. To play, you just need chips of 2 colors (which will be provided in the package) and you can draw the game board in the whiteboard.</td></tr>
<tr><td>2A</td><td>P4</td><td>Logic Links

1-4 students per copy. If you have less than 5 students, all students may play with the same copy (and not use the other one).</td></tr>
<tr><td>3A</td><td>P5</td><td>Rubik's Race</td></tr>
<tr><td>4A</td><td>P6</td><td>Dara
This is a 2 player game. There is no game box. To play, you just need chips of 2 colors (which will be provided in the package) and you can draw the game board in the whiteboard.</td></tr>
<tr><td>5A</td><td>P7</td><td>Dara
This is a 2 player game. There is no game box. To play, you</td></tr>
</table>
</td></tr>
</table>

		just need chips of 2 colors (which will be provided in the package) and you can draw the game board in the whiteboard.
6A	P8	**IQ Twist**
7A	P9	**Dara** *This is a 2 player game. There is no game box. To play, you just need chips of 2 colors (which will be provided in the package) and you can draw the game board in the whiteboard.*
8A	P10	**Lunar landing**
9A	P11	**Dog Pile**

Meeting 5: May 8, at PSCB 140 (Villa)		
YOUR TABLE	**PACKAGE**	**GAME(S)**
1B	P12	**Logic Links**
2B	P13	**Logic Links**
3B	P14	**Logic Links**
4B	P15	**Rush Hour**
5B	P16	**Mastermind**
6B	P17	**Logic Dots**
7B	P18	**Dara** *This is a 2 player game. There is no game box. To play, you just need chips of 2 colors (which will be provided in the package) and you can draw the game board in the whiteboard.*
8B	P1	**Shape by Shape**
9B	P2	**Logic Links**

Game Descriptions

You can see the rules for all games here (PDF):
https://drive.google.com/open?id=14MTdFPuZvo7flw2ch01dREZJ16n9e28s

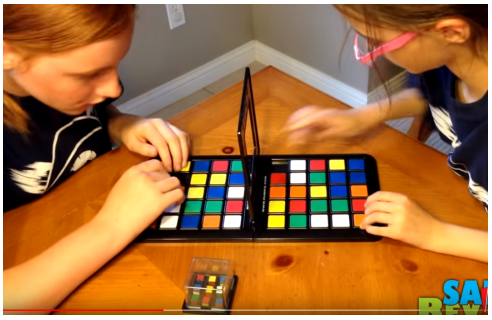
Shape by Shape

Goal: Combine the pieces to match the image on the challenge card.

This is an advanced tangram-style pattern game, made more difficult because you also fill in the frame around each shape you make. With 60 challenges, Shape By Shape is a great exercise in conceptual thinking and spatial relationships.

Video: **https://www.youtube.com/watch?v=x_Ai7trop2Q**

Rubik's Race



Goal: Be the first player to make the 3x3 center of their game board match the Scrambler (which shows a pattern).

In this game players race to complete a puzzle. Each player has a 5x5 slide puzzle with one tile missing. At the beginning of each round, the "Scrambler", a device that looks like a 3x3 Boggle grid with the cover on -- but using colored cubes instead of letters -- is shaken. Players then try to make the 3x3 center of their puzzle match the Scrambler.

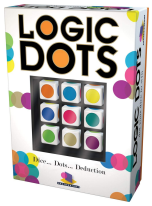
Video: **https://www.youtube.com/watch?v=PWHsHsZdBKA:**

Mastermind

Goal: one player must guess the "secret code" chosen by the other player.

The idea of the game is for one player (the code-breaker) to guess the secret code chosen by the other player (the code-maker). The code is a sequence of 4 colored pegs chosen from six colors available. The code-breaker makes a serie of pattern guesses - after each guess the code-maker gives feedback in the form of 2 numbers, the number of pegs that are of the right color and in the correct position, and the number of pegs that are of the correct color but not in the correct position - these numbers are usually represented by small black and white pegs.

Video:
https://www.amazon.com/Mastermind-Game-Strategy-Codemaker-Codebreaker/dp/B000000DMBF



Logic Dots

Goal: Find where the golden dot is, by following clues on a card.

Choose a challenge and then place nine colorful cubes into the frame according to the instructions. You only get a few clues, so you will need to use your deductive reasoning skills to solve each puzzle. Line up all the dots and your mind is golden!

Video: **https://www.youtube.com/watch?v=16ZYMCyF498**

See also: http://thepuzzleden.blogspot.com/2016/08/logic-dots-from-brainwright.html

Dog Pile

Goal: arrange the dog puzzle pieces given in the challenge card so that they fit precisely onto the grid (sometimes in 3D, so filling more than 1 “layer”.

Dog lovers will beg for this cute spatial perception puzzle. Select one of the 48 challenge cards, ranging from beginner to expert, and arrange the dog puzzle pieces so that they fit precisely onto the grid. Each colorful dog is a unique shape; you have to figure out how to nest them together. To make it even more challenging, many levels involve multiple layers!

Players: 1 or more (as a team). We suggest pairs. One can also use the 2 copies of the game and have a competition between 2 teams, selecting the same card and whoever completes the challenge first wins!

Videos: **https://www.youtube.com/watch?v=aKGqxNvl_1Y**
https://boardgamegeek.com/video/196339/brainwright/dog-pile-review-tom-vasel

Rush Hour

Goal: Slide the Red Car through the exit of the board.

First, you select a challenge card and place the cars and trucks on the traffic grid as indicated by the illustration.To Play: Slide the blocking cars and trucks in their lanes—up and down, left and right—until the path is clear for the red car to escape.Vehicles can only slide forward & backward, not sideways. You cannot lift the cars or trucks off the traffic grid surface. Stay in your lanes!

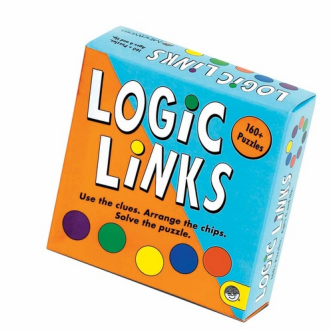
Players: 1 or more (as a team). We suggest pairs. One can also use the 2 copies of the game and have a competition between 2 teams, selecting the same card and whoever completes the challenge first wins!

Video: **https://www.amazon.com/ThinkFun-Rush-Traffic-Logic-Girls/dp/B00000DMER**

Lunar Landing

Goal: get your shuttle to the center of the board, which represents the Emergency Entry Point of your Mothership.

The idea for each turn is to get the like-colored robot to a randomly

	selected target. The trick is that once a robot starts moving, it will continue to move until a wall or another robot stops it. Therefore, players are seeking a sequence of moves for the robots that will enable them to move the required robot to the target in the fewest moves. One player calls out a number of moves that they think they can move the robot successfully to its goal and the other players have until the timer runs out to find a way to do the same thing, but in fewer moves. The person who successfully gets the robot to the target in the fewest moves gains a token and the person with the most tokens at the end of the game wins. Video: https://www.youtube.com/watch?v=2BxPr55buhM See also: http://www.sahmreviews.com/2018/02/thinkfun-lunar-landing.html **IQ Twist** Video: https://www.youtube.com/watch?v=xekKyFNATj8  **Logic Links** Goal: to solve each puzzle using the clues provided, to see where to place colored chips. Logic Links requires you to think sideways, backwards, right to left and up and down as you read the clues that lead you to the one and only correct answer! Each puzzle is comprised of a series of clues that instruct the player where to place colored chips and solve the puzzle. The set includes 166 puzzles, 32 plastic game chips, and instructions. Video: **https://www.amazon.com/Logic-Links-Puzzle-Critical-Thinking-x/dp/B000NR4BLY** **Dara** Video: https://www.youtube.com/watch?v=_O3_K7CCYFA Rules: http://www.gemsclub.org/yahoo_site_admin/assets/docs/Dara_Printable_Game.140152735.pdf

Teaching tips	• It's always a good idea to start the activity with an informal chat with students about the purpose of the game.
	  ○ Ask students what the game is about.
	  ○ You can also ask them about any strategies that they think will be good (or at the end, about those that they ended up using).
	
	• When asking questions during the game (which can encourage them to think mathematically about the moves they make), make eye contact with students. Listen carefully, with undivided attention. They will care more if they see that you care for their thinking (not just their answers).