Section 1.2, # 12: Find the solution of the given initial-value problem:

\[ \frac{dy}{dt} + ty = 1 + t, \quad y(\frac{3}{2}) = 0. \]

Solution: First, we multiply by an integrating factor \( \mu \):

\[ \mu y' + t\mu y = \mu (1 + t). \]

The integrating factor \( \mu \) must satisfy

\[ \mu' = t\mu, \]

and so

\[ \mu(t) = e^{\int t \, dt} = e^{\frac{1}{2}t^2}. \]

Now, we have

\[ (\mu y)' = e^{\frac{1}{2}t^2} (1 + t), \]

and so if we integrate with respect to time from \( \frac{3}{2} \) to \( t \), we have

\[ \int_{\frac{3}{2}}^{t} (\mu y)' \, ds = \int_{\frac{3}{2}}^{t} e^{\frac{1}{2}s^2} (1 + s) \, ds. \]

By the fundamental theorem of calculus, and because

\[ \mu \left( \frac{3}{2} \right) = e^{-\frac{9}{8}}, \quad y \left( \frac{3}{2} \right) = 0, \]

we have

\[ \mu(t)y(t) = \int_{\frac{3}{2}}^{t} e^{\frac{1}{2}s^2} (1 + s) \, ds. \]

Solving for \( y \), we have

\[ y(t) = e^{-\frac{1}{2}t^2} \int_{\frac{3}{2}}^{t} e^{\frac{1}{2}s^2} (1 + s) \, ds. \]

Section 1.4, # 6: Solve the given initial-value problem, and determine the interval of existence of the solution:

\[ t^2(1 + y^2) + 2y \frac{dy}{dt} = 0, \quad y(0) = 1. \]

Solution: First, we separate variables to obtain

\[ \frac{2yy'}{1 + y^2} = -t^2. \]

If we integrate both sides with respect to \( t \) (and use the substitution \( u = 1 + (y(t))^2, \; du = 2yy' \, dt \)) from 0 to \( t \), we get

\[ \ln |1 + y^2| - \ln |1 + y(0)^2| = -\frac{1}{3}t^3 + \frac{1}{3}0^3, \]

that is,

\[ \ln(1 + y^2) = \ln(2) - \frac{1}{3}t^3. \]

If we solve for \( y \), we have

\[ y(t) = \pm \sqrt{2e^{-\frac{1}{3}t^3} - 1}. \]
We choose the positive root because \( y(0) \geq 0 \). Therefore,

\[
y(t) = \sqrt{2e^{-\frac{1}{2}t^3} - 1}.
\]

Next, we determine for which \( t \) the solution is valid. For the square root to be real-valued, we must have

\[
e^{-\frac{1}{2}t^3} \geq \frac{1}{2},
\]

so

\[
-\frac{1}{3}t^3 \geq \ln(1/2) = -\ln 2,
\]

that is,

\[
t^3 \leq 3 \ln 2 = \ln 8.
\]

Therefore, the solution is valid on \( (-\infty, \sqrt[3]{\ln 8}) \).