SOLUTIONS

Consider the following model for forced, damped vibrations:

\[ my'' + cy' + ky = F(t) \] (1)

1. (a) (1 point) Identify the term in (1) that gives the total force. (It stems from Newton’s second law.)

**Solution:** \( my'' \)

(b) (1 point) Identify the term in (1) that deals with the spring constant.

**Solution:** \( ky \)

c) (1 point) Identify the term in (1) that deals with dampening.

**Solution:** \( cy' \)

d) (1 point) Identify the term in (1) that deals with forcing.

**Solution:** \( F(t) \)

2. (6 points) Consider the following differential equation

\[ y'' + 4y' + 4y = 4t^2 + 12t + 6. \]

Find a particular solution \( \psi \).

**Solution:** We’ll proceed by “judicious guessing.” Because the righthand side is a second-degree polynomial, we guess that \( \psi \) is also a second-degree polynomial of the form

\[ \psi(t) = a + bt + ct^2. \]

(Note: In some situations, it may be necessary to use a higher degree polynomial. I’d recommend you also try the problem \( y'' + 4y' = 4t^2 + 12t + 6 \).)

So, we have

\[
\begin{align*}
y(t) &= a + bt + ct^2 \\
y'(t) &= b + 2ct \\
y''(t) &= 2c,
\end{align*}
\]

so

\[
\begin{align*}
y'' &= 2c \\
y' &= b + 2ct \\
y' &= 4b + 8ct \\
y'' &= 4a + 4bt + 4ct^2 \\
t^2(4c) + t(12) + 1(6) &= t^2(4c) + t(8c + 4b) + 1(2c + 4b + 4a).
\end{align*}
\]

By equating like terms, we have

\[ 4 = 4c \Rightarrow c = 1, \]
$$12 = 8c + 4b = 8 + 4b \Rightarrow 4 = 4b \Rightarrow b = 1,$$

and

$$6 = 2c + 4b + 4a = 6 + 4a \Rightarrow a = 0.$$ 

Thus,

$$\psi = t + t^2.$$ 

This wasn’t asked in the problem, but the general solution to the DE is

$$y(t) = c_1 e^{-2t} + c_2 te^{-2t} + t + t^2.$$