Facts to Know:

Complex Numbers: \( a + bi \) where \( a, b \in \mathbb{R} \) real numbers \( i^2 = -1 \)

- Addition: Adding the real and imaginary parts
- Multiplication: FOIL and simplify using \( i^2 = -1 \)

Polar Form:
- \( r \) - radius from origin
- \( \theta \) - angle from positive x-axis

- Addition: Convert to rectangular form
- Multiplication: \( (r_1 e^{i \theta_1})(r_2 e^{i \theta_2}) = r_1 r_2 e^{i(\theta_1 + \theta_2)} \)

Examples:

1. Write \( i \) and \( -i \) in polar form.

\[
i = 1 e^{i \pi/2} = e^{i \pi/2} \\
-i = 1 e^{-i \pi/2} = e^{-i \pi/2}
\]

Note: There are multiple ways to represent a complex number in polar form. (Add/subtract \( 2\pi \) from the angle.)
2. Based on your answer from Question 1, what happens when we multiply by \( i \)?

\[
i = e^{\frac{i \pi}{2}}
\]

\[
i \cdot e = e^{\frac{i \pi}{2}} \cdot e = e^{i(\frac{\pi}{2} + \frac{\pi}{2})} = e^{i \pi} = -1
\]

3. Use the polar representation of \( i \) and \(-1\) to show that \( i^2 = -1 \).

\[
i = e^{\frac{i \pi}{2}}
\]

\[
i^2 = i \cdot i = e^{\frac{i \pi}{2}} \cdot e^{\frac{i \pi}{2}} = e^{i(\frac{\pi}{2} + \frac{\pi}{2})} = e^{i \pi} = -1
\]

\[-1 = 1 e^{i \pi} = e^{i \pi}
\]

4. Write \( 1 - i \) and \( \sqrt{3} + i \) in polar form and multiply.

\[
1 - i = 2 e^{-\frac{i \pi}{4}}
\]

\[
\sqrt{3} + i = 2 e^{\frac{i \pi}{6}}
\]

\[
(2 e^{-\frac{i \pi}{4}})(2 e^{\frac{i \pi}{6}}) = 4 e^{\frac{-i \pi}{12} + \frac{i \pi}{6}} = 4 e^{\frac{i \pi}{12}} = 2 \sqrt{2} e^{\frac{i \pi}{12}}
\]
\[
\frac{\pi}{6} - \frac{\pi}{4} = \frac{2\pi}{12} - \frac{3\pi}{12} = \frac{-\pi}{12}
\]