1. Convert \(i\) and 1 to polar form and show that \(i^4 = 1\).

**Solution:** Both 1 and \(i\) are on the unit circle, 1 has an angle of 0 and \(i\) has an angle of \(\pi/2\) so their polar representations are

\[
1 = 1e^{0i} \quad i = 1e^{\pi i/2}
\]

Then

\[
i^4 = \left(e^{\pi i/2}\right)^4 = e^{2\pi i} = e^{0i} = 1
\]

2. Write \(-2 + 2i\) and \(1 + \sqrt{3}i\) in polar form and multiply.

**Solution:** \(-2 + 2i\) has radius and angle

\[
r = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \quad \theta = \arctan(2/ -2) = 3\pi/4
\]

so it has representation \(2\sqrt{2}e^{3\pi i/4}\). Also \(1 + \sqrt{3}i\) has radius and angle

\[
r = \sqrt{1^2 + \sqrt{3}^2} = 2 \quad \theta = \arctan(\sqrt{3}/1) = \pi/3
\]

so the polar representation is \(2e^{\pi i/3}\). Therefore the product is

\[
\left(2\sqrt{2}e^{3\pi i/4}\right) \left(2e^{\pi i/3}\right) = 4\sqrt{2}e^{13\pi i/12}
\]

3. What is \(\frac{1}{re^{i\theta}}\) in polar form? [Hint: If \((re^{i\theta})(se^{i\phi}) = 1\) what do we need \(s\) and \(\phi\) to be?]

**Solution:** Let \(se^{i\phi}\) be the polar representation of \(1/re^{i\theta}\), so

\[
e^{i\theta} = 1 = (re^{i\theta})(se^{i\phi}) = rs e^{i(\theta + \phi)}
\]

Then we need \(rs = 1\) and \(\theta + \phi = 0\) so \(s = r^{-1}\) and \(\phi = -\theta\). Therefore

\[
\frac{1}{re^{i\theta}} = r^{-1}e^{-i\theta}
\]