MATH 120A Prep: Complex Numbers I

1. Convert i and 1 to polar form and show that $i^4 = 1$.

> **Solution:** Both 1 and i are on the unit circle, 1 has an angle of 0 and i has an angle of $\pi/2$ so their polar representations are
> $$1 = 1e^{0i} \qquad i = 1e^{\frac{\pi i}{2}}$$
> Then
> $$i^4 = \left(e^{\frac{\pi i}{2}}\right)^4 = e^{2\pi i} = e^{0i} = 1$$

2. Write $-2 + 2i$ and $1 + \sqrt{3}i$ in polar form and multiply.

> **Solution:** $-2 + 2i$ has radius and angle
> $$r = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2} \qquad \theta = \arctan(2/-2) = 3\pi/4$$
> so it has representation $2\sqrt{2}e^{3\pi/4}$. Also $1 + \sqrt{3}i$ has radius and angle
> $$r = \sqrt{1^2 + \sqrt{3}^2} = 2 \qquad \theta = \arctan\left(\sqrt{3}/1\right) = \frac{\pi}{3}$$
> so the polar representation is $2e^{\pi i/3}$. Therefore the product is
> $$\left(2\sqrt{2}e^{3\pi i/4}\right)\left(2e^{\pi i/3}\right) = 4\sqrt{2}e^{13\pi i}12$$

3. What is $\dfrac{1}{re^{i\theta}}$ in polar form? [Hint: If $(re^{i\theta})(se^{i\phi}) = 1$ what do we need s and ϕ to be?]

> **Solution:** Let $se^{i\phi}$ be the polar representation of $1/re^{i\theta}$, so
> $$e^{i0} = 1 = (re^{i\theta})(se^{i\phi}) = rse^{i(\theta+\phi)}$$
> Then we need $rs = 1$ and $\theta + \phi = 0$ so $s = r^{-1}$ and $\phi = -\theta$. Therefore
> $$\frac{1}{re^{i\theta}} = r^{-1}e^{-i\theta}$$