Facts to Know:

Equivalence Relation: An equivalence relation \( \sim \) on a set \( S \) satisfies three properties:

- Reflexive:

- Symmetric:

- Transitive:

Equivalence Class: The equivalence class of an element \( x \) is

Connection to Partitions:

- Equivalence Relation to Partition:

- Partition to Equivalence Relation:

Examples:

1. Define a relation on the set of people where \( A \sim B \) if the age of person \( A \) equals the age of person \( B \). Show this is an equivalence relation.
2. Show that the relation on \( \mathbb{Z} \) defined by \( x \sim y \) whenever \( 3 \mid (x - y) \) is an equivalence relation. What are the equivalence classes?

3. Let \( S = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \text{ are 0 or 1}\} \). Define a relation on \( S \) by \((x, y, z) \sim (u, v, w)\) if \( x + y + z = u + v + w \). Show this is an equivalence relation and write out the equivalence classes.

4. Suppose we want an equivalence relation so that the classes are the lines of slope 5 in \( \mathbb{R}^2 \). How should we define the equivalence relation?