

## MATH 120A Prep: Equivalence Relations

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1. Define a relation on the set of all people by  $A \sim B$  if and only if  $A$  is at least as tall as  $B$ . Is this an equivalence relation?
2. Define a relation on  $\mathbb{R}$  by  $x \sim y$  if  $|x| = |y|$ . Show this is an equivalence relation and list the elements of the equivalence classes  $[0]$ ,  $[1]$ , and  $[-3]$ .
3. Define a relation on  $\mathbb{R}^2$  by  $(x, y) \sim (u, v)$  if  $x^2 + y^2 = u^2 + v^2$ . Show this is an equivalence relation and describe the equivalence classes.

4. Let  $S = \{(a, b) : a, b \in \mathbb{Z}, b \neq 0\}$ . Define a relation on  $S$  by  $(a, b) \sim (c, d)$  if  $ad = bc$ . It turns out this is actually an equivalence relation. (You can prove this if you like, but it is a bit long and isn't necessary for the problem.) List some elements of the equivalence classes  $[(3, 2)]$  and  $[(-1, 5)]$ . The set of equivalence classes can be represented by a familiar set of numbers, what is it? [Hint: Write  $(a, b)$  as  $\frac{a}{b}$ ]