

MATH 120A Prep: Modular Arithmetic

Facts to Know:

Fix a positive integer n . Then $x \sim y$ if $n|(x - y)$ is an equivalence relation on \mathbb{Z} . The quotient set of equivalence classes is the set:

$$\mathbb{Z}_n =$$

Operations on \mathbb{Z}_n :

- Addition:
- Multiplication:

Well-Defined Operations: Show that different choices of representative don't affect the output of the function. If $f : \mathbb{Z}_n \rightarrow X$ we want to show that

Examples:

1. List the elements of \mathbb{Z}_4 and create an addition and multiplication table.

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2. Show that the function $f : \mathbb{Z}_5 \rightarrow \mathbb{Z}_5$ defined by $f([a]) = [a^2]$ is well-defined. Is it a bijection?

3. Let $[a]_n$ denote the equivalence class of a in \mathbb{Z}_n . Prove that the map $g : \mathbb{Z}_3 \rightarrow \mathbb{Z}_6$ defined by $g([a]_3) = [2a]_6$ is well-defined and injective but not surjective. Is $g([a]_3) = [3a]_6$ well-defined?