Examples:

1. Which proof technique would you use to show \( \sqrt{2} \) is irrational?

   Prove something is not true \( \rightarrow \) contradiction

   Setup: Assume \( \sqrt{2} \) is rational...

   Want to show: This gives a contradiction.

2. Which proof techniques would you use to show that \( n^2 \) is odd if and only if \( n \) is odd?

   Forward: If \( n^2 \) odd, then \( n \) odd.
   \[ \text{not even} \rightarrow \text{not even} \]
   \[ \# \text{Contrapositive} \]
   Assume \( n \) even.
   Conclude that \( n^2 \) is even.

   Backward: If \( n \) odd, then \( n^2 \) odd.
   \[ \text{not even} \rightarrow \text{not even} \]
   \[ \# \text{Direct} \]
   Assume \( n \) odd.
   Conclude that \( n^2 \) is odd.

3. Which proof technique would you use to show that for any positive integer \( n \), that \( n^3 - n \) is divisible by 3?

   Induction.

   Base Case: Show true for \( n = 1 \).

   Inductive Step: If this is true for \( n \), then it is also true for \( n+1 \).
4. Which proof technique would you use to show that for sets $A, B, C$ we have $(A \setminus B) \cup (C \setminus B) \subseteq (A \cup C) \setminus B$?

$x \in (A \setminus B) \cup (C \setminus B)$ is really saying $x \in A \setminus B$ or $x \in C \setminus B$ or usually implies Proof by Cases.

**Case 1:** $x \in A \setminus B$
Show $x \in (A \cup C) \setminus B$

**Case 2:** Assume $x \in C \setminus B$
Show $x \in (A \cup C) \setminus B$.

One of the two options has to happen, so the statement is true.

5. Which proof technique would you use to show that if $r$ and $s$ are rational numbers then $r + s$ is rational?

Direct proof:
Assume $r$ is rational and $s$ is rational.
Prove $r + s$ is rational.