MATH 120A Prep: Proof Techniques

Examples:

1. Which proof technique would you use to show $\sqrt{2}$ is irrational? not rational

Prove something is not true → contradiction

Setup: Assume $\sqrt{2}$ is rational...

Want to show: This gives a contradiction.

2. Which proof techniques would you use to show that n^2 is odd if and only if n is odd?

Forward: If n^2 odd, then n odd.

not even   not even

* Contrapositive.

Assume n even.

Conclude that n^2 is even.

Backward: If n odd, then n^2 odd.

not even   not even

* Direct.

Assume n odd.

Conclude that n^2 is odd.

3. Which proof technique would you use to show that for $\boxed{\text{any positive integer } n}$ that $n^3 - n$ is divisible by 3?

Induction.

Base Case: Show true for $n = 1$.

Inductive Step: If this is true for n, then it is also true for $n+1$.

4. Which proof technique would you use to show that for sets A, B, C we have $(A\backslash B)\cup(C\backslash B) \subseteq (A\cup C)\backslash B$?

$x \in (A\backslash B)\cup(C\backslash B)$ is really saying $x \in A\backslash B$ OR $x \in C\backslash B$

OR usually implies Proof by Cases.

Case 1: $x \in A\backslash B$

Show $x \in (A\cup C)\backslash B$

Case 2: Assume $x \in C\backslash B$

Show $x \in (A\cup C)\backslash B$.

One of the two options has to happen, so the statement is true.

5. Which proof technique would you use to show that if r and s are rational numbers then $r+s$ is rational?

Direct proof.

Assume r is rational and s is rational.

Prove $r+s$ is rational