MATH 130A Review: Algebra of Sets

Facts to Know

- Sets
  - \( U \) (the universal set)
  - \( \emptyset \) (the empty set)
  - Subsets

- Unions
  - \( A \cup B = \text{the collection of objects } x \text{ such that } x \text{ is an element of } A \text{ or } x \text{ is an element of } B \)

- Intersections
  - \( A \cap B = \text{the collection of objects } x \text{ such that } x \text{ is an element of } A \text{ and } x \text{ is an element of } B \)

- Complements
  - \( A^c = \text{the collection of objects } x \text{ such that } x \text{ is not an element of } A \)

- Associative laws
  - \( A \cup (B \cup C) = (A \cup B) \cup C \)
  - \( A \cap (B \cap C) = (A \cap B) \cap C \)

- Commutative laws
  - \( A \cup B = B \cup A \)
  - \( A \cap B = B \cap A \)

- Distributive laws
  - \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)
  - \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \)

- Miscellaneous facts
  - \( A \cap B = \emptyset \text{ and } A \cup B = U \implies B = A^c \)
  - \( \emptyset^c = U \)
  - \( U^c = \emptyset \)
  - \( A^c = A \)
  - \( (A \cup B)^c = A^c \cap B^c \)
  - \( (A \cap B)^c = A^c \cup B^c \)
Examples

• If the universal set is given by $U = \{1, 2, 3, 4, 5, 6\}$, and $A = \{1, 2\}$, $B = \{2, 4, 5\}$ are two subsets, find the following sets:
  o $A \cup B = \{1, 2, 4, 5\}$
  o $A \cap B = \{2\}$
  o $A^c = \{3, 4, 5, 6\}$
  o $B^c = \{1, 3, 6\}$

• If the universal set is given by $U = (-\infty, +\infty) = \mathbb{R}$, and $A = [1, 2]$, $B = [2, 5]$ are two subsets, find the following sets:
  o $A \cup B = [1, 5]$
  o $A \cap B = \{2\}$
  o $A^c = (-\infty, 1) \cup (2, +\infty)$
  o $B^c = (-\infty, 2) \cup (5, +\infty)$