MATH 134A Review: The point estimate for the population variance

1. Let $X_1, \dots, X_n$ be independent and identically distributed (discrete) random variables. Let μ be the expectation of $X_1, \dots, X_n$ and let σ^2 be the variance of $X_1, \dots, X_n$. Define $\overline{X} := \frac{X_1 + \cdots + X_n}{n}$. Prove $\mathbb{E}(\overline{X}) = \mu$.

Solution:

$$\mathbb{E}(\overline{X}) = \mathbb{E}(\frac{X_1 + \cdots + X_n}{n}) = \frac{1}{n}\mathbb{E}(X_1 + \cdots + X_n) = \frac{1}{n}(\mathbb{E}(X_1) + \cdots + \mathbb{E}(X_n)) = \frac{n\mu}{n} = \mu$$

2. Let $X_1, \dots, X_n$ be independent and identically distributed (discrete) random variables. Let μ be the expectation of $X_1, \dots, X_n$ and let σ^2 be the variance of $X_1, \dots, X_n$. Define $\overline{X} := \frac{X_1 + \cdots + X_n}{n}$. Prove $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$.

Solution:

$$\mathrm{Var}(\overline{X}) = \mathrm{Var}(\frac{X_1 + \cdots + X_n}{n}) = \frac{1}{n^2}\mathrm{Var}(X_1 + \cdots + X_n) = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$