Facts to Know:

Quantifiers:

• (For all $\forall$) The statement holds for all $x$.
• (There exists $\exists$) The statement holds for some $x$.

Negation:

• (not for all) The statement is false for some $x$.
• (not there exists) The statement is false for every $x$.

When negating a statement with quantifiers, remember to

• First, the statement that needs to hold.

• If there is more than one quantifier, handle them one at a time from left to right.

Examples:

1. Negate the following statement: Every human has an X-chromosome.

   There exists a human that does not have an X-chromosome.

2. Negate the following statement: There exists a human with a Y-chromosome.

   Every human does not have a Y-chromosome.
3. Negate the following statement: Let \( a_n \) be a sequence of real numbers and let \( L \) be a real number. For every \( \epsilon > 0 \), there exists \( N \in \mathbb{N} \) such that for all \( n \in \mathbb{N} \), if \( n \geq N \), then \( |a_n - L| < \epsilon \).

\[
\forall \epsilon > 0, \exists N \in \mathbb{N}, \forall n \in \mathbb{N}, \text{if } n \geq N, \text{ then } |a_n - L| \geq \epsilon.
\]

There exists \( \epsilon > 0 \) such that for all \( N \in \mathbb{N} \), there exists \( n \in \mathbb{N} \) such that

\[
n \geq N \text{ and } |a_n - L| \geq \epsilon.
\]