Facts to Know:

Properties of inequalities:
1. (addition) \( a < b \) \iff a + \epsilon \leq b + \epsilon \)
2. (multiplication by \( \epsilon > 0 \)) \( a < b \) \iff a \cdot \epsilon \leq b \cdot \epsilon \)
3. (multiplication by \( \epsilon < 0 \)) \( a < b \) \iff a \cdot \epsilon > b \cdot \epsilon \)

Properties of absolute value:
1. The absolute value of \( x \) is defined by \( |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \)
2. \( |x| < \epsilon \) \iff -\epsilon < x < \epsilon 
3. \( |a \cdot b| = |a| \cdot |b| \)
4. Does \( |a + b| = |a| + |b| \) generally hold? \( 0 = |1 - 1| \neq 1 + 1 = 2 \)
5. \( |a + b| \leq |a| + |b| \)

Example: Let \( n = 1, 2, \ldots \) Determine for what values of \( n \) the following holds
\[
\left| \frac{7n + 5}{3n - 4} - \frac{7}{3} \right| < \frac{1}{2020}.
\]

Solution. \((\star)\) holds \iff
\[
-\frac{1}{2020} < \frac{7n + 5}{3n - 4} - \frac{7}{3} < \frac{1}{2020}.
\]
\iff
\[
-\frac{1}{2020} + \frac{7}{3} < \frac{7n + 5}{3n - 4} < \frac{1}{2020} + \frac{7}{3}. \quad (\star \star)
\]
\iff \( n = 1 \), then \((\star)\) is false. If \( n \geq 2 \),
then $3n-4 > 0$. Thus, \((\star \star)\) holds iff

\[
\frac{-(3n-4)}{2020} + 7n - \frac{7 \cdot 4}{3} < 7n + 5 < \frac{3n-4}{2020} + 7n - \frac{7 \cdot 4}{3}
\]

iff

\[
\frac{-(3n-4)}{2020} < 5 + \frac{7 \cdot 4}{3} < \frac{3n-4}{2020}.
\]

Since $\frac{-(3n-4)}{2020} < 0 < 5 + \frac{7 \cdot 4}{3}$, then we only need to focus on:

\[
5 + \frac{7 \cdot 4}{3} < \frac{3n-4}{2020}
\]

iff

\[
9652.4 = \left\lfloor \left(5 + \frac{7 \cdot 4}{3}\right)2020 + 4 \right\rfloor / 3 < n.
\]

Thus, \((\star \star)\) holds iff $n \geq 9652.4$. □