1. Rewrite $\sqrt{2} + \sqrt{3}$ as a fraction with no radicals in the numerator.

**Solution:** We have that

$$
(\sqrt{2} + \sqrt{3}) \frac{(\sqrt{2} - \sqrt{3})}{(\sqrt{2} - \sqrt{3})} = \frac{2 - 3}{(\sqrt{2} - \sqrt{3})} = \frac{-1}{(\sqrt{2} - \sqrt{3})}.
$$

2. Rewrite $\sqrt{n + 2} - \sqrt{n - 1}$ as a fraction with no radicals in the numerator and then compute the limit as $n \to \infty$.

**Solution:** Since we notice that $\sqrt{n + 2} - \sqrt{n - 1}$ is of the form $x - y$, we will multiply by

$$
\frac{\sqrt{n + 2} + \sqrt{n - 1}}{\sqrt{n + 2} + \sqrt{n - 1}} = 1,
$$

to get

$$
(\sqrt{n + 2} - \sqrt{n - 1}) \cdot 1 = (\sqrt{n + 2} - \sqrt{n - 1}) \cdot \frac{\sqrt{n + 2} + \sqrt{n - 1}}{\sqrt{n + 2} + \sqrt{n - 1}}
$$

$$
= (\sqrt{n + 2} - \sqrt{n - 1}) \cdot \frac{(n + 2) - (n - 1)}{\sqrt{n + 2} + \sqrt{n - 1}}
$$

$$
= \frac{3}{\sqrt{n + 2} + \sqrt{n - 1}}.
$$

Notice that the trick gets rid of the radicals at the top. We then have

$$
(\sqrt{n + 2} - \sqrt{n - 1}) = \frac{3}{\sqrt{n + 2} + \sqrt{n - 1}}.
$$

Hence, $\sqrt{n + 2} - \sqrt{n - 1} \to 0$ as $n \to \infty$.

3. What is the limit of $a_n = n^{2/n}$ as $n \to \infty$?

**Solution:** We have that

$$
a_n = n^{2/n} = e^{\ln(n^{2/n})} = e^{2/n \ln(n)}.
$$

By using L’hopital’s rule, we get

$$
\lim_{n \to \infty} \frac{2 \ln n}{n} = 2 \lim_{n \to \infty} \frac{1}{n} = 0.
$$

Since the function $e^x$ is continuous, then we have

$$
\lim_{n \to \infty} a_n = \lim_{n \to \infty} e^{2/n \ln(n)} = e^{\lim_{n \to \infty} 2/n \ln(n)} = e^0 = 1.
$$