MATH 140A Review: Set builder notation in \( \mathbb{R} \)

Simplify the notation:

1. \( \mathbb{N} \cap [0, 1.2) \)

**Solution:** We have that \( x \in \mathbb{N} \cap [0, 1.2) \) if and only if \( x \in \mathbb{N} \) and \( x \in [0, 1.2) \). Hence, \( x \in \mathbb{N} \cap [0, 1.2) = 1 \).

2. \( \{ x \in \mathbb{R} : x > 2 \} \cap (-\infty, 8] \)

**Solution:** We have that \( y \in \{ x \in \mathbb{R} : x > 2 \} \cap (-\infty, 8] \) if and only if \( y \in \{ x \in \mathbb{R} : x > 2 \} = (2, \infty) \) and \( y \in (-\infty, 8] \). Thus, \( \{ x \in \mathbb{R} : x > 2 \} \cap (-\infty, 8] = (2, 8] \).

3. \( \mathbb{R}^c \)

**Solution:** We will show that the set is empty by way of contradiction. Assume that there exists a real number \( x \in \mathbb{R}^c \). Then, \( x \not\in \mathbb{R} \). This is a contradiction since \( \mathbb{R} \) is the set of all real numbers, but \( x \) is a real number not in \( \mathbb{R} \). Thus, the set \( \mathbb{R}^c \) is empty. That is, \( \mathbb{R}^c = \emptyset \).

4. \( \mathbb{Z} \cap \mathbb{Q}^c \)

**Solution:** We will show that the set is empty by way of contradiction. Assume that there exists \( x \in \mathbb{Z} \cap \mathbb{Q}^c \). Then, \( x \in \mathbb{Z} \) and \( x \in \mathbb{Q}^c \). Since \( x \in \mathbb{Q}^c \), then \( x \) is irrational. This is a contradiction since \( x \in \mathbb{Z} \), that is, \( x \) is an integer number (which is also a rational number). Thus, the set \( \mathbb{Z} \cap \mathbb{Q}^c \) is empty. That is, \( \mathbb{Z} \cap \mathbb{Q}^c = \emptyset \).