MATH 140A Review: Sequences and Series

Facts to Know:

How do we add an infinite list of numbers?

\[
\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots
\]

Answer: Take a look at the partial sums:

\[
S_1 = a_1
\]
\[
S_2 = a_1 + a_2
\]
\[
S_3 = a_1 + a_2 + a_3
\]
\[
\vdots
\]
\[
S_N = a_1 + a_2 + a_3 + \cdots + a_N = \sum_{n=1}^{N} a_n
\]

If the limit of the sequence \(\{S_N\}_{N=1}^{\infty}\) exists, then define

\[
\sum_{n=1}^{\infty} a_n = \lim_{N \to \infty} S_N = \lim_{N \to \infty} \sum_{n=1}^{N} a_n
\]

and say that the series converges. Otherwise, the series diverges.

Example: Determine if the series

\[
\sum_{n=9}^{\infty} \ln \frac{1 + \frac{1}{n}}{1 + \frac{1}{n-1}}
\]

converges? If it converges, what does the series add up to?
Facts to Know:

Let \( r \in \mathbb{R} \). The sequence \( a_n = r^n \) is called the geometric sequence.

\[
\lim_{n \to \infty} r^n = \begin{cases} 
0 & |r| < 1, \\
1 & r = 1, \\
\text{DNE} & r \leq -1, \\
\infty & 1 < r.
\end{cases}
\]

The geometric series

\[
\sum_{n=0}^{\infty} r^n = \frac{1}{1 - r},
\]

converges for \(|r| < 1\) and diverges otherwise.

Example: Determine if the following series converges. If so, what is the sum?

\[
\sum_{n=2}^{\infty} 3 \cdot \frac{1}{4^n}.
\]